Basic/Essential Course Information	
Course title	Mathematical Methods of Physics
Degree Course title	Physics
ECTS	6
Compulsory attendance	No
Course teaching language	ENGLISH

Teacher	Prof. Paolo Facchi	paolo.facchi@uniba.it

ECTS Details	Disciplinary area/broad field:	SSD	ECTS
	Characterizing	FIS/02	6

Time management and teaching activity type	Period	Year	lesson type
	1st semester	1st	Lessons (40h) Exercises (15h)

Time management	Total hours	in-class/in-lab study hours	out-of-class study hours
	150	55	95

Course calendar	Starting date	Ending date
	Last week of September	Third week of December

Syllabus	
Prerequisites	Real and complex analysis, Fourier transform, Distribution theory, Quantum mechanics.
Expected learning outcomes (according to Dublin Descriptors)	**Knowledge and understanding** The student will acquire knowledge of the advanced mathematical techniques commonly used in fundamental and applied research in physics. In particular, a knowledge of the mathematical structures of functional analysis and the theory of operators on Hilbert spaces, necessary for understanding advanced problems of modern physics. **Applying knowledge and understanding** The student will acquire knowledge of general and advanced analytical and approximation techniques for understanding quantum phenomena and solving problems in quantum mechanics and quantum field theory. **Making judgements** Within the mathematical methods of physics, the student will be able to identify the best mathematical strategy for tackling specific physical problems. **Transferable Communication skills** The student will acquire mastery of the mathematical lexicon of modern

	physics and of quantum physics. **Lifelong learning skills** The student will develop an attitude to the continuous updating of mathematical techniques and skills in physics research.
Course contents summary	Metric spaces. Banach spaces. Measure theory. Hilbert spaces. Linear operators on Hilbert spaces. Spectrum and dynamics.
Detailed syllabus	**Metric spaces.** Definition. Examples. Open sets, closed sets, neighborhoods. Topological spaces. Continuous mappings. Dense sets, separable spaces. Convergent and Cauchy sequences. Completeness. Examples. Completion of a metric space. **Banach spaces.** Vector spaces. Normed spaces. Completeness and Banach spaces. Examples: finite dimensional spaces, sequence spaces, function spaces. Bounded linear operators. Continuity and boundedness. BLT theorem. Continuous linear functionals and dual spaces. Banach space of bounded linear operators. Examples. **Introduction to measure theory.** Lebesgue integral. Sigma algebras and Borel measures. Measurable functions. Dominated and monotone convergence. Fubini theorem. Examples: absolutely continuous measure, Dirac measure, Cantor measure. Lebesgue decomposition theorem. **Hilbert spaces.** Inner product. Euclidean and Hilbert spaces. Orthogonality, Pythagorean theorem. Bessel and Cauchy-Schwarz inequalities. Triangular inequality. Parallelogram law and polarization identity. Examples. Direct sum. Projection theorem. Riesz-Fréchet lemma. Orthonormal systems and Fourier coefficients. Orthonormal bases and Parseval's relation. Gram-Schmidt orthogonalization procedure. Isomorphism with l^2. Tensor product and product bases. **Linear operators on Hilbert spaces.** C∗-algebra of bounded operators. Normal, self-adjoint, unitary and projection operators. Baire's category theorem. Uniform boundedness principle. Uniform, strong and weak convergence. Some quantum mechanics. Unbounded operators. Adjoint. Symmetric and self-adjoint operators. Examples: multiplication and derivation operators. Essentially self-adjoint operators. Fundamental criteria of self-adjointness and essentially self-adjointness. Graph, closure and inverse of an operator. Self-adjoint extensions of positive operators. Example: kinetic energy in a segment. Self-adjointness of observables. **Spectrum and dynamics.** Resolvent operator, resolvent set and spectrum. Examples: position and momentum operators. First resolvent formula and analytic properties. Neumann series. Spectrum and Weyl sequences. Spectrum and eigenvalues of the inverse. Spectrum of self-adjoint, unitary and projection operators. Projection-valued measures and resolution of the identity. Integration on PVM of bounded functions. Expectation value of the resolvent. Spectral family of a self-adjoint operator and spectral theorem. Functional calculus. Spectral projections and spectral types. Quantum dynamics and unitary evolution groups. Energy conservation. Stone's theorem.
Bibliography	- M. Reed, B. Simon, *Methods of Modern Mathematical Physics*, Vol. 1, Academic Press, New York, 1980 - G. Teschl, *Mathematical Methods in Quantum Mechanics*, American Mathematical Society, Providence, 2009 - Lecture notes

Notes	Available online at http://www.ba.infn.it/~facchi/Sito/Lectures.html
Teaching methods	Lectures and exercise sessions
Assessment methods	Oral exam; written exercise
Evaluation criteria	Capability to use techniques and solve problems introduced in the course. Adequate comprehension and global knowledge of concepts and arguments described throughout the course.