Search for evidence of cosmic ray acceleration in Supernova Remnants through the study of \( \gamma \)-ray spectra with Fermi-LAT data

Relatore:
Dott. Francesco Giordano

Laureando:
Leonardo Di Venere

Anno Accademico 2013-2014
Contents

List of Figures v
List of Tables vii
Abstract ix

1 Cosmic rays 1
1.1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1
1.2 Spectrum . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2
1.2.1 GZK cut-off . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 5
1.3 Composition . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 7
1.3.1 Chemical abundances . . . . . . . . . . . . . . . . . . . . . . . 8
1.3.2 Isotopic abundances . . . . . . . . . . . . . . . . . . . . . . . 9
1.3.3 Leptonic component . . . . . . . . . . . . . . . . . . . . . . . 11
1.3.4 Antimatter component . . . . . . . . . . . . . . . . . . . . . . 12
1.4 Propagation and confinement in the Galaxy . . . . . . . . . . . . . 13
1.4.1 Spallation mechanism . . . . . . . . . . . . . . . . . . . . . . . 14
1.4.2 Cosmic ray clocks: the case of $^{10}$Be . . . . . . . . . . . . . . 16
1.4.3 Confinement time and B/C ratio . . . . . . . . . . . . . . . . 17
1.4.4 Confinement volume for cosmic rays: the leaky box model . . 18
1.5 Acceleration mechanisms . . . . . . . . . . . . . . . . . . . . . . . . 20
1.5.1 Fermi mechanism . . . . . . . . . . . . . . . . . . . . . . . . . 20
1.5.2 Second order Fermi mechanism . . . . . . . . . . . . . . . . . 21
1.5.3 First order Fermi mechanism . . . . . . . . . . . . . . . . . . 23

2 Supernova Remnants as Cosmic-ray sources 27
2.1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 27
2.2 Stellar evolution . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 27
2.2.1 Supernovae . . . . . . . . . . . . . . . . . . . . . . . . . . . . 29
2.2.2 Supernova Remnants . . . . . . . . . . . . . . . . . . . . . . . 30
2.3 Non-thermal photon emission processes . . . . . . . . . . . . . . . 34
2.3.1 Synchrotron emission . . . . . . . . . . . . . . . . . . . . . . . 35
2.3.2 Inverse Compton scattering . . . . . . . . . . . . . . . . . . . 37
2.3.3 Bremsstrahlung radiation . . . . . . . . . . . . . . . . . . . . . 39
## List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>The “all-particle” spectrum.</td>
<td>3</td>
</tr>
<tr>
<td>1.2</td>
<td>Number of sunspots and rate of cosmic rays from neutron monitors since 1958.</td>
<td>3</td>
</tr>
<tr>
<td>1.3</td>
<td>Fluxes of nuclei in primary cosmic rays.</td>
<td>7</td>
</tr>
<tr>
<td>1.4</td>
<td>Abundances of elements observed in the cosmic rays by the Cosmic Ray Isotope Spectrometer (CRIS) compared to the Solar System abundances.</td>
<td>8</td>
</tr>
<tr>
<td>1.5</td>
<td>Boron-to-carbon ratio as a function of kinetic energy per nucleon as measured by different experiments.</td>
<td>10</td>
</tr>
<tr>
<td>1.6</td>
<td>Electron plus positron spectrum in CRs.</td>
<td>12</td>
</tr>
<tr>
<td>1.7</td>
<td>Positron fraction in CRs.</td>
<td>13</td>
</tr>
<tr>
<td>1.8</td>
<td><em>Leaky box</em> propagation model.</td>
<td>18</td>
</tr>
<tr>
<td>1.9</td>
<td>Second order Fermi acceleration mechanism.</td>
<td>22</td>
</tr>
<tr>
<td>1.10</td>
<td>First order Fermi acceleration mechanism.</td>
<td>23</td>
</tr>
<tr>
<td>2.1</td>
<td>Tycho and Kepler SNRs.</td>
<td>31</td>
</tr>
<tr>
<td>2.2</td>
<td>Crab nebula.</td>
<td>31</td>
</tr>
<tr>
<td>2.3</td>
<td>SNR evolution scheme.</td>
<td>33</td>
</tr>
<tr>
<td>2.4</td>
<td>Feynman diagrams for Inverse Compton Scattering.</td>
<td>38</td>
</tr>
<tr>
<td>2.5</td>
<td>Feynman diagrams for bremsstrahlung.</td>
<td>39</td>
</tr>
<tr>
<td>2.6</td>
<td>Experimental p-p cross sections, as a function of proton momentum, compared to the modelled p-p cross section.</td>
<td>41</td>
</tr>
<tr>
<td>2.7</td>
<td>Empirical inelastic p-p cross sections, as a function of proton momentum, compared to the modelled total p-p inelastic cross section.</td>
<td>42</td>
</tr>
<tr>
<td>2.8</td>
<td>Gamma-ray spectra produced by protons with a power-law spectrum in kinetic energy through the $\pi^0$ decay.</td>
<td>43</td>
</tr>
<tr>
<td>2.9</td>
<td>Gamma-ray spectra of IC 443 and W44 as measured with the Fermi LAT.</td>
<td>45</td>
</tr>
<tr>
<td>2.10</td>
<td>Gamma-ray spectrum of Supernova Remnants Cassiopeia A as measured with the Fermi-LAT.</td>
<td>46</td>
</tr>
<tr>
<td>2.11</td>
<td>Gamma-ray spectrum of Supernova Remnants RX J1713.7-3946 as measured with the Fermi LAT.</td>
<td>47</td>
</tr>
<tr>
<td>2.12</td>
<td>Cartoon of possible scenario for a SNR interacting with a molecular cloud.</td>
<td>48</td>
</tr>
<tr>
<td>3.1</td>
<td>Schematic illustration of the Fermi-LAT apparatus.</td>
<td>50</td>
</tr>
</tbody>
</table>
3.2 Table summarising the predefined quantities associated to each photon event. ........................................... 54
3.3 Schematic representation of the Pass 8 reconstruction technique. .......................................................... 55
3.4 Expected acceptance of the instrument with the Pass 8 event reconstruction, compared to the latest P7 Reprocessed performance. .................................................. 55
3.5 Effective area of the IRF P7REP_V15. ........................................................................................................ 57
3.6 Acceptance of the IRF P7REP_V3. ............................................................................................................. 57
3.7 Point spread function of the IRF P7REP_V15. ............................................................................................ 58
3.8 Energy resolution of the IRF P7REP_V15. .................................................................................................... 58
4.1 Counts map obtained using P8 data. ............................................................................................................. 70
4.2 Spectral Energy Distribution obtained through the likelihood analysis using P7REP and P8 Fermi data. .................................................................................................................. 74
4.3 Diffuse normalisations for the P7REP analysis. .......................................................................................... 75
4.4 Diffuse normalisations for the P8 analysis. ................................................................................................. 76
4.5 Energy spectrum of the Earth Limb model. ................................................................................................. 76
4.6 SED obtained through the different fits performed by alternatively fixing or fitting the diffuse models for P7REP analysis. ..................................................................................................... 77
4.7 SED obtained through the different fits performed by alternatively fixing or fitting the diffuse models for P8 analysis. ..................................................................................................... 77
4.8 Spectral Energy Distribution obtained using P7REP and P8 data compared to the previous published data. ..................................................................................................................... 78
4.9 Residual map obtained from the P7REP analysis. ..................................................................................... 79
4.10 Residual map obtained from the P8 analysis. ............................................................................................ 80
4.11 TS map obtained from the P7REP analysis. ............................................................................................... 81
4.12 TS map obtained from the P8 analysis. ....................................................................................................... 81
4.13 Enlarged view of the TS map obtained from the P7REP analysis. ........................................................... 82
4.14 Enlarged view of the TS obtained from the P8 analysis. ......................................................................... 82
4.15 a ........................................................................................................................................................... 85
4.16 Differential proton flux adopted as injection spectrum to fit the γ-ray flux of Tycho SNR. ............................. 86
4.17 Spectral Energy Distribution of Tycho SNR from radio to TeV energy range. .................................................. 87
4.18 Spectral Energy Distribution of Tycho SNR in the MeV-TeV energy range. .................................................... 88
4.19 γ-ray flux of Tycho SNR described with a leptonic model. .......................................................................... 91
A.1 Synchrotron spectrum emitted by a single electron. .................................................................................. 96
## List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>Data selection for the analysis with Pass 7 reprocessed and Pass 8 data.</td>
<td>69</td>
</tr>
<tr>
<td>4.2</td>
<td>Spectral and spatial models adopted in the P7REP and P8 analysis.</td>
<td>71</td>
</tr>
<tr>
<td>4.3</td>
<td>Results of the fit on the full energy range for P7REP and P8 analysis.</td>
<td>73</td>
</tr>
</tbody>
</table>
Abstract

After more than one hundred years from the discovery of cosmic rays (CRs) by Victor Hess in 1912, the questions about the origin of these charged particles are still unsolved. Many observations have been performed since then, obtaining a great number of data which now have to be put in the right place to solve the puzzle of cosmic rays.

The measurements of the spectrum and composition of CRs revealed that this “radiation” consists of charged particles which reach incredibly high energies, up to $10^{20}$ eV. A detailed study gave much information about their propagation in the Galaxy and about the possible sources which can accelerate particles to such a high energy. In 1949, Enrico Fermi proposed a mechanism through which particles could be accelerated, which requires only the presence of a strong shock wave. We know that our Galaxy is full of these shock waves, deriving from the explosion of Supernovae, which are the final stage of the evolution of massive stars. Nowadays, this is still the most accepted acceleration mechanism and Supernova Remnants (SNRs), which are the remaining products of the Supernova explosion, are the most probable candidates to be source of the galactic cosmic rays. This hypothesis is known as SNR paradigm for CRs.

Unfortunately, the CR direct observation cannot give information about the source, since charged particles are deflected by the galactic magnetic field and all the directional information of the CR is lost during its propagation. As a consequence, modern experiments aim to observe the phenomena happening at the source through the observation of the electromagnetic spectrum, from the radio band to the very high energy gamma-rays. In particular, charged particles accelerated at the source interact with the environment and inevitably emit radiation, which can be detected pointing directly at the source.
Fermi-LAT is one of these experiments and has among its scientific goals the study of astrophysical objects in the γ-ray spectrum, such as Supernova Remnants. The work of this thesis is set in this frame and was developed within the Fermi-LAT collaboration. The relation between CR acceleration and CR sources was deeply analysed, focusing the attention on the study of one Supernova Remnant, Tycho SNR. This SNR represents a good case to test the acceleration theory and look for evidence of the acceleration of cosmic rays up to the knee of CR spectrum around $10^{15}$ eV, which corresponds to a break in the CR spectrum, since it is a young SNR in which the energy of the shock is still close to its maximum values.

The thesis is divided into four chapters. In Chapter 1, the main features of cosmic rays will be described, in order to highlight what we do know and what is still unknown.

In Chapter 2, a description of the features of Supernova Remnants will be provided, in order to understand why they are considered good candidates to accelerate cosmic rays. In the second part of the chapter, the attention will be focused on the relation between the acceleration mechanism and the photon emission, in order to understand the role of the photon measurements to study cosmic rays. Finally, a review of the most recent detection of SNRs in the γ-rays will be provided.

In Chapter 3, the Fermi-LAT experiment will be described, from the apparatus to the techniques used to analyse data. A particular attention will be given to those part of the analysis which have been developed in this thesis.

Finally, Chapter 4 will be dedicated to the presentation of the Fermi-LAT data analysis performed and to the results obtained. In the second part, the description of an interpretation model will be provided in order to obtain information about the CR acceleration.
Chapter 1

Cosmic rays

1.1 Introduction

Cosmic rays (CRs) are charged particles coming from the outer space. They were discovered by Victor Hess in 1912, who noticed that the ionizing radiation observed at the surface of the Earth, whose origin was initially attributed to the natural radioactivity of the Earth, increased with altitude, proving that this “radiation” came from the high. Later observations showed that this “radiation” was made of charged particles, with energy up to few $10^{20}$ eV, an incredibly high value if compared to the energies reached in the modern accelerators. During its next run in 2015, the Large Hadron Collider at CERN will reach a center-of-mass energy of 14 TeV, accelerating protons up to $7 \text{ TeV} = 7 \cdot 10^{12} \text{ eV}$, which is 8 order of magnitudes lower than the one observed in cosmic rays. Nonetheless, man-made particle accelerators are essential for studies of rare processes thanks to the detailed description of the single event and to the high statistics produced.

Cosmic rays are classified as primary or secondary. The particles directly produced and accelerated at the source are defined as Primary Cosmic Rays. They are mostly made of protons, helium nuclei and electrons, as well as carbon, oxygen, iron, and other nuclei which are synthesized in stars. These particles are accelerated up to the very high energies we measure and are injected in the environment around the source propagating to Earth. During this process they interact with the medium they come across and produce other particles, called Secondary Cosmic Rays. Nuclei such as lithium, beryllium, and boron (which are not abundant end-products of stellar nucleosynthesis) are part of this category. Antiprotons and
positrons are also in large part secondary, even though recent measurements sug-
gested that a small fraction of these particles may be primary, raising a lot of new
questions about cosmic ray origin.

These considerations are valid for particles coming from the outer space. Measure-
ments at the Earth’s surface must take into account that particles interact with
the atmosphere, producing hadronic and electromagnetic showers, mostly made of
muons coming from the pion decay in the first case and of positrons, electrons and
$\gamma$-rays in the second.

1.2 Spectrum

Figure 1.1 shows the energy spectrum of cosmic rays, which is the number of par-
ticles per unit time and unit solid angle incident on a unit area surface orthogonal
to the direction of observation. The spectrum is well described by a power-law
distribution over a wide energy range, from few hundreds MeV up to about a hun-
dred EeV. The differential energy spectrum has been multiplied by $E^{2.6}$ in order
to display the features of the steep spectrum that are otherwise difficult to discern.

For energy below 1 GeV, the spectrum presents a cut-off relative to the power-law
distribution, due to the solar effects on charged particles. In fact, during the pe-
riods of high solar activity, the charged plasma emitted by the Sun, called solar
wind, interacts with the incoming CRs, preventing their propagation to the Earth
and reducing the observed flux. Conversely, the CR flux reaches its maximum
during the periods of low solar activity. This phenomenon is known as solar mod-
ulation and has a cycle of about 11 years. The evidence of this phenomenon is
obtained through the measurements of the intensity of cosmic rays through neu-
tron monitors, which detect the number of neutrons generated in the interaction
of primary CRs with the Earth’s atmosphere. An (anti-)correlation is observed
between the measured rate of neutrons and the number of sunspots, whose pres-
ence was first noticed by Galileo Galilei in the XVII century and is a visible sign
of the high solar activity (see figure 1.2).

For energies above 10 GeV, particles are not affected by the solar wind and their
spectrum follows a power-law:

$$F(E) = E^{-\alpha},$$  \hspace{1cm} (1.1)
Chapter 1. Cosmic rays

Figure 1.1: The “all-particle” spectrum as a function of energy-per-nucleus from air shower measurements. [1]

Figure 1.2: Number of sunspots and rate of cosmic rays from neutron monitors since 1958 [2]. The anti-correlation of these two quantities with a period of 11 years gives a strong evidence of the solar modulation of cosmic rays.
where $\alpha$ represents the spectral index of the distribution. Its value changes significantly in two points of the spectrum. The first break, known as knee, occurs at an energy around $10^{15}$ eV, where the spectral index changes from a value of 2.7 to a value of 3. For energies above $10^{19}$ eV, which corresponds to the second break in the spectrum and is known as ankle, the spectral index becomes again 2.7.

The features observed in the spectrum may give information about the origin of cosmic rays. Particles with energy below $10^{18}$ eV are thought to be of galactic origin. In this picture, the knee could reflect the fact that most of cosmic ray accelerators in the Galaxy reach their maximum energy of acceleration between $10^{15}$ eV and $10^{18}$ eV, causing a break in the spectrum. Actually, the spectrum produced by a single source with a maximum energy $E_{\text{max}}$ would present an exponential cut-off for energies above $E_{\text{max}}$. However, the combination of different exponential cut-off energies in the range $10^{15}$ eV - $10^{18}$ eV would result in a steepening of the power-law spectrum. The Kascade-Grande experiment reported the observation of a second steepening of the spectrum near $8 \cdot 10^{16}$ eV, with evidence that this structure is accompanied by a transition to heavy primaries, which can be explained by the fact that the maximum energy of acceleration is related to the confinement of particles in the acceleration region and then scales with the charge of the particle.

Supernova remnants are considered good candidates for the acceleration of cosmic rays up to energies close to the knee. As it will be shown later, the Fermi acceleration mechanism predicts that a shock, like the one originating in a Supernova explosion, can accelerate charged particles, producing a power-law spectrum.

Effects of propagation and confinement in the Galaxy also need to be considered, since they modify the observed spectrum of cosmic rays with respect to the one produced at the source.

Concerning the ankle, one possibility is that it is the result of a higher energy population of particles overtaking a lower energy population, for example an extragalactic flux beginning to dominate over the galactic flux. In this case, the most probable candidates to accelerate cosmic rays up to these energies are the Active Galactic Nuclei (AGN), which are very far galaxies that emit a bright radiation in the entire electromagnetic spectrum.
1.2.1 GZK cut-off

The hypothesis of the extragalactic origin of cosmic rays with energy above the ankle, called Ultra-High Energy Cosmic Rays (UHECR), is reinforced by the observation of a cut-off in the spectrum for energy above $5 \cdot 10^{19}$ eV, known as Greisen-Zatsepin-Kutzmin (or GZK) cut-off. This phenomenon appears when the energy of cosmic-ray protons is sufficiently high to activate the photo-pion and photo-pair production processes.

If a proton is bombarded with high energy $\gamma$-rays, pions are created:

\begin{align}
\gamma + p & \rightarrow n + \pi^+, \\
\gamma + p & \rightarrow p + \pi^0 \rightarrow p + \gamma + \gamma, \\
\gamma + p & \rightarrow n + N\pi.
\end{align}

The photon threshold energy for these reactions is approximately $\epsilon_t = 200$ MeV and the cross section is about 250 microbarn = $2.5 \cdot 10^{-32}$ m$^{-2}$.

All the free space in the Universe is permeated by photons of the Cosmic Microwave Background (CMB), which is the thermal radiation assumed to be the left over of the Big Bang, and therefore cosmic rays cannot escape from it. The CMB spectrum is described by a black body spectrum with a temperature of approximately 2.7K, corresponding to an average energy of $\epsilon_0 = 6 \cdot 10^{-4}$ eV, which is much lower than the threshold energy $\epsilon_t$. However, in the rest frame of a cosmic-ray proton with Lorentz factor $\gamma$, this energy becomes

$$\epsilon = \epsilon_0 \gamma (1 + \beta \cos \theta),$$

where $\theta$ is the angle between the photon and the proton directions. If the Lorentz factor $\gamma$ is sufficiently high, this value can exceed the threshold $\epsilon_t$, activating the photo-pion production. The threshold value for the Lorentz factor of the proton, obtained in the limit $\beta \rightarrow 1$ and $\cos \theta = 1$, is $\gamma_t = \epsilon_t/(2\epsilon_0) = 1.7 \cdot 10^{11}$, corresponding to an energy $E = \gamma_t m_p = 1.7 \cdot 10^{20}$ eV. The proper calculation involves integration over the Planck spectrum of the CMB and over all angles and in this case the proton threshold energy for the photo-pion production process decreases to $5 \cdot 10^{19}$ eV.
The mean free path for a single scattering is \( \lambda = (\sigma_{\text{p} \pi} N_{\text{ph}})^{-1} \). Taking \( N_{\text{ph}} = 5 \cdot 10^8 \text{cm}^{-3} \) for the CMB and \( \sigma_{\text{p} \pi} = 2.5 \cdot 10^{-32} \text{m}^{-2} \), then \( \lambda \approx 10^{23} \text{m} \approx 3 \text{Mpc} \), which, assuming that the proton travels at the speed of light, corresponds to a propagation time of \( 10^7 \) years. The energy of the pion created in this process is \( \gamma m_{\pi} \), so that the fractional loss of energy of the cosmic ray proton is \( \Delta E/E \approx m_{\pi}/m_p \approx 1/10 \). The total mean free path for the cosmic ray proton to lose all its energy corresponds to a propagation time of \( 10^8 \) years.

A similar calculation can be carried out for the photo-pair production process:

\[
\gamma + p \rightarrow p + e^+ + e^-.
\] (1.6)

The threshold for this process is 1.02 MeV, about 200 times less than the photopion production mechanism, corresponding to a threshold proton energy of about \( 10^{18} \text{eV} \). The cross-section for this process in the ultra-relativistic limit is \( \sigma_{\text{pair}} = 10^{-30} \text{m}^{-2} \), which is 40 times larger than \( \sigma_{\text{p} \pi} \). However, each photo-pair production event removes only a fraction of \( 10^{-3} \) of the energy of the proton. As a result, the propagation time for the protons to lose all their energy is \( 2.5 \cdot 10^9 \) years, which is 25 times longer than the previous case. As a consequence, this process is less important and it just results in a distortion of the particle spectrum down to energies of about \( 10^{18} \text{eV} \).

In conclusion, if the extragalactic origin of UHECR is correct, a cut-off in the cosmic-ray energy spectrum is expected at about \( 5 \cdot 10^{19} \text{eV} \) for protons, which cannot have originated further than about 30 Mpc from our Galaxy. On the contrary, a galactic population of cosmic rays with the same energy would not suffer of the same effect, since the propagation distances would be much shorter than the value of \( \lambda \). In fact, at such a high energy, cosmic rays are not confined in the Galaxy by the Galactic magnetic field and propagate for distances of the order of the dimension of our Galaxy, which has a diameter of 30 kpc.

As can be seen in figure 1.1, very recent measurements of the cosmic-ray spectrum at energies above \( 10^{19} \text{eV} \) by some experiments like the Auger experiment seem to show this cut-off in the spectrum, supporting the extragalactic origin of UHECR.
Figure 1.3: Fluxes of nuclei of the primary cosmic rays as a function of the kinetic energy per nucleus. [1]

1.3 Composition

The observations of cosmic rays at the top of the atmosphere show that about 98% of the particles are protons and nuclei, while the remaining 2% are electrons. Of the protons and nuclei, about 87% are protons, 12% are helium nuclei and the remaining 1% are heavier nuclei.

Figure 1.3 shows that the energy spectrum of each component is consistent with the overall spectrum up to energies close to the knee. The study of the relative abundances of these elements gives information about the acceleration mechanisms at the source and the propagation mechanisms in the Interstellar Medium (ISM).
Figure 1.4: Abundances of elements observed in the cosmic rays by the Cosmic Ray Isotope Spectrometer (CRIS) compared to the Solar System abundances. Abundances normalized to the value of 1000 for Si.  

1.3.1 Chemical abundances

The chemical abundances of elements in cosmic rays give important information about their origin and their propagation in the Galaxy. Figure 1.4 represents the relative abundances of elements observed in cosmic rays, compared to the ones observed in the Solar System.

The most important features which appear in this plot are the following:

- the odd-even effect in the relative stabilities of the nuclei, which is due to the pairing term in the binding energy of nuclei, is evident in both curves;
- the abundance peaks at carbon, nitrogen and oxygen and at the iron group are present in both curves, supporting the idea that these elements do have origin directly in the sources, thanks to processes typical of stellar evolution;
- in cosmic rays there is an overabundance of light elements (Li, Be, B) and of elements with atomic number just less than iron.
The last point can be attributed to the *spallation* mechanism. During their propagation to the Earth, primary cosmic ray nuclei collide with the gas in the interstellar medium and are fragmented, resulting in the production of nuclei with atomic and mass number less than those of primary nuclei. In particular, spallation of *medium* group elements (carbon, nitrogen and oxygen) produces light elements like lithium, beryllium and boron, while spallation of iron results in nuclei like manganese, chromium and vanadium.

Spallation mechanism can be treated in a formal way using the diffusion-loss equation of cosmic rays, as it will be shown in section 1.4.

### 1.3.2 Isotopic abundances

In addition to overall chemical abundances, isotopic abundances are available for a number of species, in particular for the lightest elements $^1$H, $^2$H, $^3$He and $^4$He.

Most of helium was synthesised in the Hot Big Bang through the p-p chain. $^2$H and $^3$He, which are synthesised in the p-p chain as well, are much less stable and are destroyed much more easily. They are present in greater abundances in the cosmic rays ($\approx 10^{-5}$) than they are in the interstellar medium ($\approx 10^{-2}$). This aspect can be attributed to spallation reactions between the four species considered, providing an independent check of the spallation models.

Another important information which can be obtained from the isotopic abundances regards the propagation or confinement time of cosmic rays in the Galaxy, which is the time that a cosmic ray spends in the Galaxy before escaping from it.

Some of the species created in spallation reactions are radioactive and hence they should decay before reaching the Solar System. However, if their decay time is comparable to the propagation time, an appreciable fraction of these isotopes will survive. The comparison of their abundance with respect to the stable isotopes gives information about how much time the unstable nucleus has spent in the Galaxy, giving an estimate of the propagation time. One of the most famous of these *cosmic ray clocks* is the isotope $^{10}$Be, which has a radioactive half-life of $1.5 \cdot 10^6$ years. Detailed calculations of the radioactive to stable isotopes ratio for different species give a unique confinement time of $(15 \pm 1.6)$ Myr. Also in this case, the diffusion-loss equation of cosmic rays, gives a formal explanation of this mechanism.
A third aspect that can be analysed thanks to relative abundances is the energy dependence of this confinement time. It has been observed that, for energy above 1 GeV, the chemical composition of cosmic rays decreases with increasing energy with a power-law relation $E^{-0.6}$.

Figure 1.5 shows the measurements of the boron to carbon ratio as a function of the kinetic energy of the particle, which is proportional to the escape path length and to the escape time of the heavier species (boron in this case). Therefore:

$$\tau(E) \propto E^{-\delta},$$

with $\delta \approx 0.3 - 0.6$. As a consequence, the diffusion coefficient which appears in the diffusion-loss equation depends also on energy, being $D(E) \sim 1/\tau(E) \sim E^8$. The diffusion-loss equation gives again a description of this process.

The fact that the confinement time decreases with increasing energy means that high energy particles manage to escape easier from the Galaxy, resulting in a lower observed flux at high energy. As a consequence, the acceleration and the propagation of cosmic rays must take place at different times, otherwise the secondary
to primary CR ratio should be constant with energy.

However, propagation and acceleration are not completely uncorrelated. For example, in the shock acceleration theory from a supernova explosion, which will be described in section 1.5, the acceleration process does happen during the supernova remnant expansion in the interstellar medium. Moreover, the irregularities of the magnetic field have an important role both in the acceleration of particles and in their propagation.

### 1.3.3 Leptonic component

The leptonic component of cosmic rays is made essentially of electrons and positrons. The spectrum of electrons and positrons incident at the top of the atmosphere is expected to steepen by one power of $E$ at an energy of $\approx 5$ GeV because of strong radiative energy loss effects in the Galaxy.

Figure 1.6 shows the electron plus positron spectrum multiplied by a factor $E^3$ measured by different experiments, confirming an overall trend of $E^{-3}$. Some deviation from this spectrum are evident from the ATIC experiment, which measured an excess of electrons over propagation model expectations at energies of 300-800 GeV. The Fermi-LAT experiment measured a not-entirely flat spectrum without confirming the peak of the ATIC excess at 600 GeV. The HESS imaging atmospheric Cherenkov array also measured the electron flux above 400 GeV, finding indications of a cutoff above 1 TeV, but no evidence for a pronounced peak.

Figure 1.7 shows the positron fraction, defined as the ratio of positron flux divided by the electron plus positron flux, as measured by the PAMELA and AMS-02 satellite experiments. These experiments have a magnetic spectrometer which allows the charge discrimination. The measurements show an increase of the positron fraction above 10 GeV instead of the expected decrease from propagation models (heavy black line in the plot), confirming earlier hints seen by the HEAT balloon-borne experiment.

The structure in the electron spectrum, as well as the increase in the positron fraction, may be related to contributions from individual nearby sources (supernova remnants or pulsars) emerging above a background suppressed at high energy by synchrotron losses. Other explanations have invoked propagation effects or dark
matter decay/annihilation processes. The significant disagreement in the ratio below 10 GeV is attributable to differences in charge-sign dependent solar modulation effects present near the Earth at the time of the measurements.

### 1.3.4 Antimatter component

Antiprotons have been detected in cosmic rays. The ratio of antiprotons to protons is \( \approx 2 \cdot 10^{-4} \) at 10-20 GeV, in agreement with the antiproton production in CR propagation, giving no evidence of a significant primary component of antiprotons and strengthening the hypothesis of a galactic origin of the positron excess.

No antihelium or antideuteron has been found in the cosmic radiation. The best measured upper limit on the ratio antihelium/helium is currently approximately \( 10^{-7} \). The upper limit on the flux of antideuterons around 1 GeV/nucleon is approximately \( 2 \cdot 10^{-4} (\text{m}^2 \text{ s sr GeV/nucleon})^{-1} \).
1.4 Propagation and confinement in the Galaxy

The interaction of charged particles with the interstellar medium (scattering with target gas and interaction with magnetic field) results in a diffusive motion of particles, which can be described by a diffusion equation.

The transport equation gives the chance to describe both acceleration and propagation. For a species $i$ of cosmic rays the equation can be written as follows:

$$\frac{\partial N_i}{\partial t} = \nabla \cdot (D_i \nabla N_i) - \frac{\partial}{\partial E} [b_i N_i(E)] - \nabla \cdot u N_i(E) + Q_i - p_i N_i + \frac{v \rho}{m} \sum_{k>i} \int \frac{d\sigma(E,E')}{dE} N_k(E')dE',$$  \hspace{1cm} (1.8)

where $N_i(E, \mathbf{x}, t)dE$ represents the density of particles of species $i$ at position $\mathbf{x}$, at time $t$, with energy between $E$ and $E + dE$. 

**Figure 1.7:** Positron fraction measured by different experiments. Latest measurements are taken by the experiments AMS-02 and PAMELA. The heavy black line is a model of pure secondary production and the three thin lines show three representative attempts to model the positron excess with different phenomena. Green: dark matter decay; blue: propagation physics; red: production in pulsars. [1]
Chapter 1. Cosmic rays

The first term in the right side of the equation (1.8) represents the diffusion of cosmic rays. \( D \) is the diffusion coefficient and can be expressed in the form

\[
D = \frac{1}{3} \lambda_D v, \tag{1.9}
\]

where \( \lambda_D \) is the diffusion mean free path and \( v \) if the particle velocity. The second term represents the energy loss (for example for synchrotron emission). The coefficient \( b(E) \) has different expressions according to the type of process in consideration. The third term represents the convection process at velocity \( u \). The fourth term \( Q_i(x, t) dE \) is the source term and represents the number of particles of species \( i \) per \( \text{cm}^3 \) injected at position \( x \) and time \( t \) with energy between \( E \) and \( E + dE \). Finally, the last two terms represent the loss rate and production rate of species \( i \), respectively. The coefficient \( p_i \) includes all the loss contributions, such as the escaping time and the spallation losses. The last term includes the production rate for species \( i \) deriving from the spallation of all heavier species.

### 1.4.1 Spallation mechanism

The diffusion-loss equation gives information about the expected rates of particles due to the spallation mechanism. A simplified version of the equation (1.8) can be considered (see [6]), neglecting the diffusion and energy loss terms. Furthermore, if the rate equation for species which are not injected at the source, such as the light elements lithium, beryllium and boron, is considered, the source term \( Q_i \) can also be neglected. The equation obtained is:

\[
\frac{\partial N_i}{\partial t} = -\frac{N_i}{\tau_i} + \sum_{k>i} P_{ki} \frac{N_k}{\tau_k}, \tag{1.10}
\]

where \( \tau_i \) is the spallation lifetime for species \( i \) and \( P_{ki} \) is the probability that a nucleus of species \( i \) is created in an inelastic collision involving the destruction of a nucleus \( k \).

This equation can be re-written in terms of the path length \( \xi = \rho x = \rho vt \) (measured in \( \text{kg m}^{-2} \)), where \( \rho \) is the gas density and \( v \) is the particle velocity:

\[
\frac{\partial N_i(\xi)}{\partial \xi} = -\frac{N_i(\xi)}{\xi_i} + \sum_{k>i} \frac{P_{ki}}{\xi_k} N_k(\xi). \tag{1.11}
\]
In order to perform a simplified calculation, the rate equation for a heavy and a light species is considered, including in the heavy species the elements that are fragmented by spallation (for example the medium o M group of elements carbon, nitrogen and oxygen) and in the light species the ones that are generated as a result of the spallation process (the light o L group of elements lithium, beryllium and boron). Initially, there are no particles in the L group at \( \xi = 0 \). The differential equations to describe the L and M groups are:

\[
\frac{dN_M(\xi)}{d\xi} = -\frac{N_M(\xi)}{\xi_M}, \tag{1.12}
\]

\[
\frac{dN_L(\xi)}{d\xi} = -\frac{N_L(\xi)}{\xi_L} + \frac{P_{ML}}{\xi_M} N_M(\xi). \tag{1.13}
\]

Integrating equation (1.12),

\[
N_M(\xi) = N_M(0) \exp(-\xi/\xi_M). \tag{1.14}
\]

Multiplying equation (1.13) by an integrating factor \( \exp(\xi/\xi_L) \) and integrating:

\[
\frac{d}{d\xi} \left[ \exp(\xi/\xi_L) N_L(\xi) \right] = \frac{P_{ML}}{\xi_M} \exp(\xi/\xi_L - \xi/\xi_M) N_M(0), \tag{1.15}
\]

\[
\frac{N_L(\xi)}{N_M(\xi)} = \frac{P_{ML} \xi_L}{\xi_L - \xi_M} \left[ \exp(\xi/\xi_L - \xi/\xi_M) - 1 \right]. \tag{1.16}
\]

In this simplified treatment, average values of \( \xi_L, \xi_M \) and \( P_{ML} \) are adopted (see Table 10.1a in [6]). From the inelastic cross-sections for the M elements, the value of \( P_{ML} \) is found to be 0.28, while \( \xi_M = 60 \text{ kg m}^{-2} \) and \( \xi_L = 84 \text{ kg m}^{-2} \). From the relative abundances measurements in cosmic rays, \( \frac{N_L(\xi)}{N_M(\xi)} = 0.25 \). Inserting these values into (1.16), the typical path length through which the M elements would have to pass to create the observed abundance ratio of the L to M elements is \( \xi = 48 \text{ kg m}^{-2} \), of the same order of magnitude as the mean free path of the M elements, which is hardly surprising.

The same type of calculation can be performed for the production of \(^3\text{He} \) by the spallation of \(^4\text{He} \) in the interstellar gas.

There are obviously some discrepancies with experimental data, coming from the fact that this is a simplified model. These discrepancies can be removed if a distribution of path lengths is considered instead of assuming that all the high...
energy particles traverse the same amount of matter in reaching the Earth. In this case the complete diffusion-loss equation must be considered.

### 1.4.2 Cosmic ray clocks: the case of \(^{10}\text{Be}\)

The radioactive species created in spallation reactions can be used to “date” the samples of cosmic rays observed near the Earth.

The most famous example of these clocks is the radioactive isotope of beryllium \(^{10}\text{Be}\), which has a life-time \(\tau_r\) of the same order of magnitude as the cosmic ray escape time \(\tau_e\). From spallation reactions, the expected fraction of the \(^{10}\text{Be}\) with respect to its stable isotopes \(^{7}\text{Be}\) and \(^{9}\text{Be}\) is about 10%. If the escape time \(\tau_e\) is much longer than \(\tau_r\), this fraction should be much less than 10% and should be of the order of \(\tau_r/\tau_e\).

This statement can be derived using the simplified version of the diffusion-loss equation (1.10). In this case, a loss term for the propagation time \(-N_i/\tau_e\) must be added in the equation.

Defining the production rate of species \(i\) as

\[
C_i = \sum_{k > i} \frac{P_{ki}}{\tau_k} N_k
\]

and considering the steady state solution of the equation \((dN_i/dt = 0)\), the equation becomes:

\[
- \frac{N_i}{\tau_e(i)} + C_i - \frac{N_i}{\tau_{spal}(i)} = 0 \Rightarrow N_i = \frac{C_i}{1/\tau_e(i) + 1/\tau_{spal}(i)}.
\]

For the radioactive isotope, a decay loss term must be included \(-N_i/\tau_r\), where \(\tau_r\) is the characteristic decay time:

\[
- \frac{N_i}{\tau_e(i)} + C_i - \frac{N_i}{\tau_{spal}(i)} - \frac{N_i}{\tau_r(i)} = 0 \Rightarrow N_i = \frac{C_i}{1/\tau_e(i) + 1/\tau_{spal}(i) + 1/\tau_r(i)}.
\]

So, the steady state ratio of the \(^{10}\text{Be}\) to \(^{7}\text{Be}\) isotopes is:

\[
\frac{N(^{10}\text{Be})}{N(^{7}\text{Be})} = \frac{1/\tau_e(^{10}\text{Be}) + 1/\tau_{spal}(^{10}\text{Be})}{1/\tau_e(^{10}\text{Be}) + 1/\tau_{spal}(^{10}\text{Be}) + 1/\tau_r(^{10}\text{Be})} \frac{C(^{10}\text{Be})}{C(^{7}\text{Be})}.
\]
If the time-scale for the destruction of the beryllium isotopes by spallation is much greater than their escape times $\tau_{\text{spal}} \gg \tau_e$, a simpler expression is obtained:

$$\frac{N^{(10}\text{Be})}{N^{(7}\text{Be})} = \frac{1}{\tau_e^{(7}\text{Be})} \frac{C^{(10}\text{Be})}{C^{(7}\text{Be})}. \quad (1.21)$$

Measurements of this ratio led to an escape time of 10 Myr.

### 1.4.3 Confinement time and B/C ratio

As discussed in section [1.3.2](#), the boron-to-carbon ratio decreases with increasing energy.

The simplest interpretation of the energy dependence of secondary to primary ratio is that the path length of the primary particles through interstellar gas changes with energy. Suppose that $\xi_e(E) = \xi_0(E/E_0)^{-\alpha}$, being $\alpha$ a positive number to obtain the requested trend of the path length.

Starting from equations (1.12) and (1.13), including a loss term of the form $-N_L/\xi_e(E)$ in equation (1.13) and considering the steady state solution for this equation ($dN_L/dt = 0$), one obtains:

$$-\frac{N_L}{\xi_e(E)} + \frac{P_{\text{ML}}}{\xi_M} N_M - \frac{N_L}{\xi_L} = 0. \quad (1.22)$$

In the high energy limit, the escape path length is much less than the spallation path length, $\xi_e \ll \xi_L$, and the solution of equation (1.22) becomes

$$\frac{N_L(\xi)}{N_M(\xi)} = P_{\text{ML}} \frac{\xi_e(E)}{\xi_M}. \quad (1.23)$$

Since $P_{\text{ML}}$ and $\xi_M$ are independent on energy, the energy dependence of the ratio of secondary to primary particles is the same as the one of the escape path length $\xi_e(E)$ and of the escape time $\tau_e$, which is proportional to $\xi_e(E)$. Hence, the measurements of the boron-to-carbon ratio shown in figure [1.5](#) are a direct evidence of the decrease of the escape time with increasing energy.


1.4.4 Confinement volume for cosmic rays: the leaky box model

The value of the confinement time found with the previous considerations is not compatible with a free propagation of the relativistic particles in the Galaxy. If the confinement volume had dimension of 10 kpc (order of magnitude of the dimension of the Galactic disk) and if relativistic particles propagate at velocity close to that of light, they would escape from the Galaxy in about $3 \times 10^4$ years. Furthermore, their distribution on the sky would be highly anisotropic, since most of the flux would come from the center of the Galaxy, in contrast with the observed isotropy of cosmic rays. In fact, a small degree of anisotropy is observed ($\approx 10^{-3}$), but it is compatible with the anisotropy resulting from the diffusive motion of cosmic rays. A higher degree of anisotropy has been observed for UHECR, which are much less confined in the Galaxy. A possible association of these cosmic rays with some extragalactic sources is under investigation.

For this reason, cosmic rays are supposed to be confined in the Galaxy in a certain volume by the galactic magnetic field ($B \approx 2 \mu G$). A hint of this aspect comes from the fact that the values of the energy densities of the magnetic field ($\rho_B = B^2/2\mu_0 \approx 0.2$ eV cm$^{-3}$) and cosmic rays ($\rho_{CR} \approx 1$ eV cm$^{-3}$) are of the same order of magnitude, suggesting that the two components are related to each other.
Chapter 1. Cosmic rays

The confinement of cosmic rays in the Galaxy is often described by the leaky box model. Particles are supposed to freely propagate in a box with a horizontal dimension equal to the diameter of the Galactic disk \((\approx 30 \text{ kpc})\) and a height \(h\) of 3-5 kpc (see figure 1.8) and be reflected by the boundaries of this box. At each reflection, the particle will have a constant probability of escaping from the Galaxy \(\tau_e^{-1} \ll c/h\). The diffusion term in equation (1.8) is replaced by a loss term \(-N_i/\tau_e\). Neglecting collision processes and convection, the solution of the equation for a source term \(Q(E,t) = N_0(E)\delta(t)\) is:

\[
N(E,t) = N_0(E)\exp(-t/\tau_e). \tag{1.24}
\]

In this picture, \(\tau_e\) can be interpreted as the average time that a cosmic ray spends in the confinement volume.

If a steady source term \(Q_i(E)\) is considered for the species \(i\) and including only a loss term due to interaction with the interstellar gas, equation (1.8) becomes:

\[
\frac{\partial N_i}{\partial t} = -\frac{N_i}{\tau_e} + Q_i - \frac{N_i}{\tau_i}. \tag{1.25}
\]

The steady-state solution of this equation \((\frac{\partial N_i}{\partial t} = 0)\) is:

\[
N_i(E) = \frac{Q_i(E)\tau_e}{1 + \tau_e/\tau_i}. \tag{1.26}
\]

For protons, it is observed that \(\tau_e \ll \tau_i\) for all energies and equation (1.26) reduces to

\[
N(E) = Q(E)\tau_e(E). \tag{1.27}
\]

From cosmic ray measurements we know that both \(N(E)\) and \(\tau_e(E)\) have a power-law dependence on energy with spectral index \(-2.7\) and \(-0.6\) respectively. As a result, also the source spectrum \(Q(E)\) must follow a power-law shape:

\[
Q(E) \propto E^{-\alpha}, \tag{1.28}
\]

where the spectral index \(\alpha\) is approximately 2.1.

In conclusion, in order to reproduce the observed spectrum, cosmic ray sources must accelerate particles up to very high energies and produce a power-law spectrum with spectral index close to the value of 2. In next section, an acceleration
mechanism (proposed by Fermi in 1949) satisfying these conditions will be described.

1.5 Acceleration mechanisms

Acceleration theories have a fundamental role in the description of a “standard” model for cosmic rays, since they must account for the observed energies and spectra.

The First order Fermi mechanism, also known as diffusive shock acceleration mechanism, is based on the strong shock waves generated in the Galaxy, such as the ones originating from a Supernova explosion. The most important feature of this mechanism is that the energy gain for a particle crossing the shock wave is directly proportional to the velocity of the shock, resulting in a power-law spectrum with spectral index close to -2.

1.5.1 Fermi mechanism

Be $\Delta E = \xi E$ the energy earned by a particle after a collision and $P_{\text{esc}}$ the escape probability from the acceleration region after each collision. After $n$ collisions, a particle with initial energy $E_0$ will have an energy $E_n = E_0(1 + \xi)^n$, with a probability of being in the acceleration region equal to $(1 - P_{\text{esc}})^n$. The number of collision necessary to reach an energy $E$ is obtained inverting the first relation: $n = \ln(E/E_0)/\ln(1 + \xi)$. The number of particles with energy greater than $E$ will be proportional to the probability of having a particle with energy greater than $E$:

$$N(\geq E) \propto \sum_{m=n}^{\infty} (1 - P_{\text{esc}})^m = \frac{(1 - P_{\text{esc}})^n}{P_{\text{esc}}}.$$  \hspace{1cm} (1.29)

Using the expression of $n$ obtained previously:

$$N(\geq E) \propto \frac{1}{P_{\text{esc}}} \left( \frac{E}{E_0} \right)^{-\gamma},$$ \hspace{1cm} (1.30)

where

$$\gamma = \ln \left( \frac{1}{1 - P_{\text{esc}}} \right)/\ln(1 + \xi).$$ \hspace{1cm} (1.31)
In conclusion, the differential energy spectrum in obtained differentiating equation (1.30):

\[ N(E)dE \propto E^{-1-\gamma}dE. \]  

(1.32)

This formula reproduces the power-law spectrum necessary for the acceleration of cosmic rays and it is independent on the features of the accelerator. The only parameter depending on the accelerator is \( \xi \). So the value of the spectral index \( \gamma \) changes with the model adopted to describe the process.

### 1.5.2 Second order Fermi mechanism

In the second order Fermi mechanism, particles are assumed to be accelerated thanks to stochastic collisions with the clouds (and in particular with the magnetic field irregularities inside them, also called magnetic mirrors) moving isotropically in the interstellar medium.

Suppose that a particle with initial energy \( E \) and a magnetic mirror moving with velocity \( V \) collide forming an angle \( \theta \) between the particle trajectory and the direction perpendicular to the mirror surface (see figure 1.9). The mirror is supposed to be infinitely massive, so that it does not change velocity during the collision. In the frame of reference of the mirror, which corresponds with the center of mass frame of reference, the particle has a total energy

\[ E' = \gamma_V (E + Vp\cos\theta), \]  

(1.33)

where \( \gamma_V = (1 - V^2/c^2)^{-1/2} \) is the Lorentz factor of the mirror. The component of the momentum of the particle perpendicular to the mirror is given by

\[ p'_x = p' \cos\theta' = \gamma_V (p \cos\theta + \frac{VE}{c^2}). \]  

(1.34)

In this frame of reference the energy of the particle does not change during the collision \( E'_{\text{in}} = E'_{\text{fin}} \), while \( p'_{x,\text{in}} \) changes sign, \( p'_{x,\text{in}} = -p'_{x,\text{fin}} \).

Going back to the laboratory frame of reference and using equations (1.33) and (1.34):

\[ E'' = \gamma_V (E' + Vp' \cos\theta') = \gamma_V (E' + Vp'_x) = \gamma^2_V E \left[ 1 + \frac{2Vv\cos\theta}{c^2} + \left( \frac{V}{c} \right)^2 \right], \]  

(1.35)
where \( v \) is the initial velocity of the particle in the laboratory frame of reference, so that \( p_x/E = v \cos \theta/c^2 \).

Expanding to second order in \( \beta_V = V/c \).

\[
\Delta E = E'' - E = E \left( \frac{2 \beta_V v \cos \theta}{c} + 2 \beta^2_V \right).
\] (1.36)

The angular dependence can be eliminated taking an average over a random distribution of the pitch angle \( \theta \). The probability of encounters taking place at an angle \( \theta \) is proportional to the relative velocity of approach of the particle and the cloud, namely, from figure 1.9, \( V + v \cos \theta \) for a head-on collision and \( V - v \cos \theta \) for a following collision. Therefore, the probability can be written as \( V + v \cos \theta \) with \( 0 < \theta < \pi \). Since the particles are relativistic, \( v \approx c \) and the probability is proportional to \( (1 + V/c \cos \theta) = (1 + \beta_V \cos \theta) \).

The energy gain becomes:

\[
\left\langle \frac{\Delta E}{E} \right\rangle = 2 \beta_V \left[ \int_{-1}^{1} \frac{x(1 + \beta_V x)dx}{1 + \beta_V x} \right] + 2 \beta^2_V = \frac{8}{3} \beta^2_V.
\] (1.37)

So, the calculation showed that the average increase in energy is \textit{second-order} in \( \beta_V \).

This mechanism presents some problems. First, the velocities of the interstellar clouds are small compared to the velocity of light (\( \beta_V \leq 10^{-4} \)). Furthermore, the mean free path for the scattering of cosmic rays in the interstellar medium is of the order of 0.1 pc and so the number of collisions would amount to about a few per year, resulting in a very slow gain of energy by the particles. This means that
particles may lose the energy they gain by ionisation before being accelerated by the next cloud. Actually, this problem is present in all acceleration mechanisms and is known as injection problem. If the acceleration mechanism is to be effective, the rate of energy gain must be greater than the rate of energy losses by ionisation.

Second, there is nothing in the theory to estimate the value of the spectral index, since it depends on the characteristics of the acceleration region. On the contrary, as it will be shown in next section, first-order Fermi mechanism predicts a power-law spectrum with spectral index equal to $-2$, very close to the value required by the cosmic ray observations.

### 1.5.3 First order Fermi mechanism

First order Fermi mechanism involves the interaction of the particles with a shock wave propagating in a diffuse medium. A flux of high energy particles is assumed to be present both in front of and behind the shock front. The particles are assumed to be propagating at speeds close to that of light and so the velocity of the shock is very much less than those of the high energy particles. The high energy particles scarcely notice the shock at all since its thickness is normally very much smaller than the gyroradius of the high energy particle. Because of scattering and turbulent motions on either side of the shock wave, when the particles pass through the shock in either direction, they are scattered and their velocity distribution rapidly becomes isotropic in the frame of reference of the moving fluid on either side of the shock.
A typical shock wave propagates with a velocity $U$ of $10^4$ km s$^{-1}$, which is much higher than the sound speed ($\approx 10$ km s$^{-1}$). Be $\rho_1$ and $\rho_2$ the gas densities in the upstream and the downstream respectively. It is often convenient to transform into the frame of reference in which the shock front is at rest, so that the upstream gas flows into the shock at velocity $v_1 = U$ and leaves the shock with a downstream velocity $v_2$. The equation of continuity requires that mass is conserved through the shock, so:

$$\rho_1 v_1 = \rho_1 U = \rho_2 v_2.$$  \hfill (1.38)

In the case of a strong shock, $\rho_1/\rho_2 = (\gamma + 1)/(\gamma - 1)$, where $\gamma$ is the ratio of specific heat capacities of the gas. Taking $\gamma = 5/3$ for a monoatomic of fully ionised gas, $\rho_1/\rho_2 = 4$ and so $v_2 = v_1/4$ (figure 1.10a).

Consider the frame of reference in which particles in the upstream are at rest (figure 1.10b). The shock advances through the medium at velocity $U$ but the gas behind the shock travels at a velocity $(3/4)U$ relative to the upstream gas. When a high energy particle crosses the shock front, it obtains a small increase of energy or order $\Delta E/E \propto U/c$. The particles are then scattered in the region behind the shock front so that their velocity distributions become isotropic with respect to that flow.

Now consider the opposite process of the particle diffusing from behind the shock to the upstream region (figure 1.10c). When the particles cross the shock front, they encounter gas moving towards the shock front with the same velocity $(3/4)U$ as the previous case, which means that the particle receives the same small amount of energy $\Delta E$.

This is the main difference with respect to the second-order mechanism. Every time particles cross the shock, they increase their energy, that means that there are no collision in which the particles lose energy.

To evaluate the average increase in energy, consider a particle crossing the shock front from the upstream to the downstream. The gas on the downstream side approaches the particle at a velocity $V = (3/4)U$ and so, performing a Lorentz transformation, the particle’s energy when it passes into the downstream region is

$$E' = \gamma_V (E + p_x V),$$  \hfill (1.39)

where the $x$-coordinate is taken perpendicular to the shock.
The shock is assumed to be non-relativistic, so \( V \ll c \) and \( \gamma V \approx 1 \), while the particles are relativistic, so \( E = pc \) and \( p_x = (E/c) \cos \theta \). Therefore,

\[
\frac{\Delta E}{E} = \frac{V}{c} \cos \theta = \beta V \cos \theta. \tag{1.40}
\]

The probability that the particles which cross the shock arrive within the angles \( \theta \) to \( \theta + d\theta \) is proportional to \( \sin \theta \, d\theta \) and the rate at which they approach the shock front is proportional to the \( x \)-component of their velocities, \( c \cos \theta \). Therefore the probability of the particle crossing the shock is proportional to \( \sin \theta \, d\cos \theta \). So, defining \( x = \cos \theta \) the average energy gain is

\[
\left\langle \frac{\Delta E}{E} \right\rangle = \beta V \int_0^1 x \, x \, dx = \frac{2}{3} \beta V. \tag{1.41}
\]

The particle’s velocity is then randomised without energy loss by scattering in the downstream region. Then, it recrosses the shock and gains another fractional increase in energy \((2/3)\beta V\). Therefore, in a round trip across the shock and back again, the fractional energy increase is, on average,

\[
\left\langle \frac{\Delta E}{E} \right\rangle = \frac{4}{3} \beta V. \tag{1.42}
\]

Consequently, recalling the definition of the quantity \( \xi \),

\[
\xi = \frac{4}{3} \beta V = \frac{4}{3} \frac{3 U}{c} = \frac{U}{c} \tag{1.43}
\]

in one round trip.

The other parameter which has to be evaluated is the escape probability \( P_{\text{esc}} \). According to classical kinetic theory, the number of particles crossing the shock is \((1/4)Nc\) where \( N \) is the number density of particles. This is the average number of particles crossing the shock in either direction. Downstream, however, because the particles are isotropic, they are swept away from the shock at a rate \( NV = (1/4)NU \). Thus, the fraction of the particles lost per unit time is \((1/4)NU/(1/4)Nc = U/c\). Since the shock is assumed to be non-relativistic, only a very small fraction of the particles is lost per cycle. Thus,

\[
P_{\text{esc}} = U/c. \tag{1.44}
\]
Substituting (1.43) and (1.44) in (1.31) and taking into account that \( U \ll c \),

\[
\gamma = -\frac{\ln(1 - P_{esc})}{\ln(1 + \xi)} = -\frac{\ln(1 - U/c)}{\ln(1 + U/c)} \approx -\frac{-U/c}{U/c} = 1. \tag{1.45}
\]

Therefore, the differential energy spectrum from equation (1.32) becomes

\[
N(E)dE \propto E^{-2}dE, \tag{1.46}
\]

which is the result we have been seeking.

In conclusion, first-order Fermi acceleration mechanism is much more efficient than the second-order mechanism, since the energy gain is linear in the velocity of the shock. Furthermore, the predicted spectrum is a simple power-law with unique value for the spectral index, which is also close to the value required to account for the observed cosmic ray spectrum. The only requirements are the presence of strong shock waves and that the velocity vectors of the high energy particles are randomised on either side of the shock. For this reason, supernova remnants are very good candidates to be acceleration sites for cosmic rays.
Chapter 2

Supernova Remnants as Cosmic-ray sources

2.1 Introduction

As already pointed out in the previous chapter, Supernova Remnants (SNRs) are very good candidates to be source of accelerated particles thanks to the shock wave originating in a Supernova explosion. If a star is sufficiently massive, it ends its life as a Supernova, generating a shock wave which propagates in the interstellar medium. The expanding ejected material and the material shocked along the way constitute the Supernova Remnant.

Since supernovae are relatively rare (2-3 per century in a typical spiral galaxy like our own), SNRs also provide a good way to study the local population of supernovae and reveal details about the explosion mechanism that are difficult to obtain from studying supernovae directly.

2.2 Stellar evolution

The stellar evolution can be well described through the observation of the Hertzsprung-Russell (H-R) diagram, which represents the absolute magnitude\(^1\) of a star versus

\[ m = m_0 - 2.5 \log \left( \frac{L}{L_0} \right) \]

where \( L \) is the luminosity of the star and \( m_0 \) and \( L_0 \) are the values of the magnitude and the luminosity of a reference star respectively. The absolute magnitude is defined as the magnitude that the star

---

\(^1\)The apparent magnitude is defined by the equation \( m = m_0 - 2.5 \log (L/L_0) \), where \( L \) is the luminosity of the star and \( m_0 \) and \( L_0 \) are the values of the magnitude and the luminosity of a reference star respectively. The absolute magnitude is defined as the magnitude that the star
its effective temperature, or equivalently its color or spectral type\(^2\).

Most of stars, after their formation, occupy a position in the so called main sequence of the H-R diagram: the higher the temperature the lower the magnitude. Massive stars have high luminosities and high temperatures and burn their fuel more rapidly than the smaller stars. At equilibrium the gravitational force of the mass of the star is balanced by the fusion reactions in the core of the star, which consist in the production of one helium nuclei from four protons. The fusion reaction can go through two possible cycles. The first one involves the production of hydrogen isotopes deuterium and tritium, while the second involves the production of carbon, nitrogen and oxygen nuclei (CNO cycle). In massive stars \((M > 5 - 8M_\odot)\), being \(M_\odot\) the mass of the Sun, when the hydrogen in the star core is not sufficient to balance the gravitational force any more, heavier elements are formed through nuclear fusion and the star expands (red giant stage) losing the outer layers of gas. The fusion processes end when the iron is synthesised, since the reactions of production of elements heavier than iron are not exothermic.

Finally, gravitational force prevails and the star collapses. If the final mass of the star is lower than the Chandrasekhar limit \((1.4M_\odot)\), the degeneration pressure\(^3\) generated by the electrons balances the gravitational forces and the star becomes a white dwarf. If this condition is not satisfied, i.e. the final mass of the star exceeds the Chandrasekhar limit, the star will end its life as a Supernova. The electron degeneration pressure is not sufficient to balance the gravitational collapse and electrons and protons interact through the inverse beta decay, creating a hard core of neutrons. If the mass of the star is not too high \((M < 2.5M_\odot)\), the degeneration pressure of neutrons will prevent the star from a further collapse and a (neutron star) will be formed (otherwise a black hole will be created). The collapsing star “bounces” on this hard neutron star and creates a shock wave that propagates far from the star (supernova explosion).

This shock satisfies the conditions of the first-order Fermi acceleration mechanism and therefore can be the source of the acceleration of cosmic rays.

\(^2\)Stars are classified according to their color, which corresponds to an average wavelength of the radiation emitted, and are divided in 7 spectral classes. Going from the lower (higher) temperature (wavelength) to the higher (lower), they are: M, K, G, F, A, B and O.

\(^3\)The degeneration pressure is a quantum mechanical effect due to the Pauli exclusion principle. When fermions are compressed in a small space, their must occupy states with increasing energy, until electrons become relativistic, saturating the energy levels. The effect is that the compression cannot go further certain limits and a pressure is generated.
2.2.1 Supernovae

Supernovae are divided into two broad categories, depending on the explosion process: core collapse supernovae and thermonuclear supernovae. An old classification of supernovae, proposed by Minkowski in 1941, is based on the observed spectra: Type I supernovae do not show hydrogen absorption in their spectra, while Type II do. Type II supernovae are invariably core collapse supernovae, while Type I supernovae can be either core collapse or thermonuclear. The thermonuclear explosions are associated with spectroscopic class Type Ia, which have Si absorption lines in their spectra.

Core collapse supernovae mark the end of the lives of massive stars \((M > 8M_0)\). In these stars different layers containing the products of the consecutive burning stages are expected, with heavier elements in the core. The creation of the iron-group core, which lasts about a day, is the beginning of the end of the star, as no energy can be gained from nuclear fusion of iron. The core collapses into a neutron star, and for the most massive stars into a black hole. Most of the gravitational energy liberated \((E \approx GM^2/R_{ns} \approx 10^{53} \text{ erg})\), with \(R_{ns}\) the neutron star radius) is in the form of neutrinos. This has been confirmed with the detection of neutrinos from SN1987A by the Kamiokande [7] and Irvine-Michigan-Brookhaven [8] water Cherenkov neutrino detectors. A fraction of about \(2 \cdot 10^{51} \text{ erg}\) is deposited in the outer layers which are expelled at a velocity of approximately \(10^4 \text{ km/s}\) (supernova explosion).

Thermonuclear supernovae (Type Ia) are thought to originate from the explosions of white dwarfs, i.e. the explosion energy originates from explosive nuclear burning, rather than from gravitational energy liberated during the collapse of a stellar core. It has been observed that Type Ia supernovae have almost the same brightness, which makes them excellent distance indicators. This is in line with the idea that all Type Ia supernovae are explosions of similar objects, i.e. white dwarfs with masses close to the Chandrasekhar limit. There is no direct observational evidence that Type Ia progenitors are white dwarfs, but the fact that only Type Ia supernovae can occur among old stellar populations indicates that massive stars cannot be their progenitors. White dwarfs close to the Chandrasekhar mass limit

\[\text{This aspect was at the basis of the analysis of the relation between distance of astrophysical objects and their cosmological red-shift (which is a wavelength shift due to the expansion of the Universe), which gave an evidence that the Universe is accelerating (dark energy) (see for example [9]). For this discovery Riess, Perlmutter and Schmidt were awarded the Nobel Prize for Physics in 2011.}\]
are very likely Type Ia progenitors, since their high density is ideal for a “nuclear fusion bomb”. Once a nuclear reaction in the core is triggered, it will result in an explosion. It is clear that white dwarfs have to increase matter in order to reach the Chandrasekhar limit, so thermonuclear supernovae must occur in a binary system, such as two white dwarfs or a white dwarf with either a main sequence star or an evolved companion.

2.2.2 Supernova Remnants

SNR classification

In principle, the classification of SNRs could follow the one adopted for Supernovae. However, it is often difficult to determine the Supernova origin of a given SNR. There are some indicators which give hints about their origin. For example, the presence of a neutron star at the center of the SNR is a clear sign that the SNR must have a core-collapse origin. Furthermore, the position of the SNR in the Galaxy gives other information. Since core-collapse SNe originate from massive stars which stay in the main sequence for a shorter period, they must be located in a star forming region. On the contrary, if the SNR is located high above the Galactic plane it will probably have a Type Ia origin. Such is the case, for example, of SN 1006.

Because the supernova origin of SNRs is often difficult to establish, SNRs have an own classification, mostly based on their morphology. Traditionally, this classification recognises three classes: shell-type SNRs, plerions, and composite SNRs.

The shell-type SNRs are characterized by a limb brightened shell, which is created by a shell of shocked heated plasma originating from the interstellar medium swept by the shock wave. The shell is usually clearly visible in the radio and X-ray bands, due to the non-thermal synchrotron emission of relativistic electrons deflected by the magnetic field. This aspect will be discussed in section 4.3. Two of the most famous examples of these SNRs are Tycho and Kepler, originating from the supernova explosion of 1572 and 1604 respectively (figures 2.1).

In the case of a core-collapse Supernova, a rapidly rotating neutron star, called pulsar, is expected at the center of SNR. The pulsar will lose energy according to its rotational period and will produce a wind of relativistic electrons and positrons, which terminates in a shock, where the electrons and positrons are accelerated to
ultra-relativistic energies. These particles diffuse away from the shock creating a nebula of relativistic particles which emit synchrotron radiation from the radio to the soft X-ray bands, and inverse Compton scattering in the GeV - TeV band. Such a nebula is named pulsar wind nebula. The most famous pulsar wind nebula is the Crab Nebula, associated with the historical supernova of 1054 (figure 2.2). Since the nebula is bright at the center and does not show a shell, these SNRs are called filled center SNRs or plerions, with the name plerion deriving from the Greek word pleres, which means “full”.

Finally, energetic pulsars with ages less than 20000 years are expected to have blown a pulsar nebula while they are still surrounded by the SNR shell. One
expects then a radio and X-ray morphology that consists of a pulsar wind nebula surrounded by a shell and are classified as composite SNRs. In fact, it is still puzzling why a young object like the Crab Nebula does not show a SNR shell.

Most of the already classified SNRs are shell-type SNR, due to the fact that they are much easier to recognise. One of the most complete catalogs of SNRs was produced by Dave Green in 1984 [10], in which he collected all the known SNRs, summarising their main features, such as the position in the sky, the extension, the characteristics of the spectrum in different energy bands and some possible associations with objects observed at different wavelengths. As it will be discussed in section 2.3, since SNRs are expected to accelerate cosmic rays, \( \gamma \)-rays will be produced by accelerated particles in the MeV - TeV energy range. For this reason, the Fermi-LAT collaboration is developing a SNR catalog [11], to collect all the observed sources which can be associated to a known (or not known) SNR. At this time, 13 SNRs have been detected and published by the Fermi-LAT collaboration, but other candidates are being analysed. The work developed in this thesis is set in this context and aims to get information about the cosmic rays acceleration through the \( \gamma \)-ray observation of SNRs.

**SNR evolution**

The evolution of SNRs is usually divided in four phases:

I. the *ejecta dominated* phase of free-expansion phase in which the mass of the supernova ejecta, \( M_{ej} \), is greater than the swept-up mass, \( M_{sw} \), and the evolution of the SNR is the same as the one of the accelerated expansion of the material expelled by the SN;

II. the *Sedov-Taylor* phase or *adiabatic* phase, in which \( M_{sw} \) > \( M_{ej} \) and a shock wave propagates in the ISM and a reverse shock propagates backwards towards the center of the SNR (see figure 2.3). The radiative energy losses are not energetically important, hence the name of *adiabatic* phase;

III. the pressure-driven, or *snowplough* phase, in which radiative cooling becomes energetically important and the velocity of the shock drops. The material inside the shock joins the one outside, creating a dense shell;

IV. the merging phase, in which the shock velocity and temperature behind the shock become comparable to the turbulent velocity and temperature of the
interstellar medium, respectively, and the shock merges in the interstellar medium.

Although these discrete phases provide a useful framework of the evolution of SNRs, it should be kept in mind that it is an oversimplification, and the phase of an individual SNR is not always that easily labeled. Moreover, different parts of a SNR may be in different phases. For example, the SNR RCW 86 has radiative shocks in the south-west (phase III), whereas in the north-east it has very fast, non-radiative, shocks (phase I). This is probably due to the complexity of the medium in which the SNR is developing.

In the literature one also often finds designations for SNRs like young, mature and old. These designations do not have a very precise meaning, but a general guideline is that young SNRs are less than 1000 - 2000 years old and are in phase 1 or early in phase II, mature SNRs are in late phase II, or early phase III, whereas the label old SNRs is usually given to the very extended structures associated with SNRs in phase IV.
2.3 Non-thermal photon emission processes

The study of SNRs can give detailed information about the shock propagating in the interstellar medium, which in principle is able to accelerate particles up to very high energies through the first-order Fermi mechanism, as already discussed in section 1.5. Since accelerated charged particles are deflected by the Galactic magnetic field during their propagation to Earth, direct observation at the source cannot be performed. However, charged particles interact with the SNR environment and produce photons from radio to TeV energy range, which can be directly detected.

The main non-thermal emission mechanisms are:

- synchrotron emission, due to high energy electrons deflected in the magnetic field;
- inverse Compton scattering of high energy electrons on low energy photons from the Cosmic Microwave Background (CMB), from infrared dust emission and from starlight;
- bremsstrahlung radiation of high energy electrons accelerated in the Coulomb field generated by charged particles (electrons, protons and ions) of the gas surrounding the remnant;
- decay of neutral resonances, produced in the interaction of high energy protons with target protons and ions present in the gas.

The study of the cross sections of these processes allows a prediction of the observed photon flux from radio band to TeV $\gamma$-rays, once an injection spectrum for the accelerated particles is assumed, which in the simplest case is a simple power-law spectrum in momentum. In the following sections, a description of the emission processes will be given, and the specific photon emissivity $Q_\gamma$ (number of photons emitted per unit volume in unit time per unit energy) will be calculated per each process. In order to evaluate the photon flux at the Earth, the emissivity must be multiplied by the volume of the emitting region (which corresponds to the volume of the SNR) and divided by $4\pi d^2$, where $d$ is the distance of the SNR:

$$F_\gamma = Q_\gamma \frac{V_{SNR}}{4\pi d^2}.$$  (2.1)


2.3.1 Synchrotron emission

The synchrotron emission occurs when electrons are deflected by a magnetic field. The spectrum of the emitted photons goes from the radio to the X-ray energy band, according to the strength of the magnetic field and to the electron energy.

The power emitted by a relativistic particle of charge \( e \) with an acceleration \( \vec{a} \) is given by the generalized Larmor formula:

\[
P_e = \frac{2e^2}{3c^3} \gamma^4 \left[ \gamma^2 a^2_\parallel + a^2_\perp \right]
\]  

(2.2)

Since there is no electric field acting on the particle and neglecting any other force in the direction of motion (\( a_\parallel = 0 \)), the acceleration is given by the relativistic Lorentz force:

\[
\vec{F} = \frac{d}{dt}(\gamma m \vec{\nu}) = \frac{e}{c} \vec{\nu} \times \vec{B}
\]

(2.3)

which implies:

\[
F_\perp = \gamma ma_\perp = \frac{e\nu_\perp}{c} B \Rightarrow a_\perp = \frac{e\nu_\perp B}{\gamma mc}
\]

(2.4)

The resulting trajectory will have an helical shape, with a typical radius (Larmor radius) obtained by setting \( a_\perp = v^2_\perp/r_L \) and a corresponding relativistic gyration frequency given by \( \nu_B = \nu_\perp/(2\pi r_L) \):

\[
r_L = \frac{v^2_\perp}{a_\perp} = \frac{\gamma mc\nu_\perp}{eB}
\]

(2.5)

\[
\nu_B = \frac{eB}{2\pi\gamma mc} = \frac{\nu_L}{\gamma}
\]

(2.6)

where \( \nu_L \) is the Larmor frequency, i.e. the gyration frequency for non-relativistic particles.

However, the typical frequency of the synchrotron emission does not correspond to the revolution frequency of the particles. Since electrons are relativistic, in the laboratory frame of reference the emission is collimated in a cone of angular aperture of \( \approx 2/\gamma \) degrees. Hence photons are emitted in the direction of the observer only for a small fraction of time and the observed period of the rotation of the electron must be multiplied by the factor \( 2/\gamma \). Furthermore, due to the relativistic nature of the electrons, the time intervals are compressed of a factor \( (1 - \beta) \approx 1/(2\gamma^2) \). In the end, the typical synchrotron frequency will be multiplied...
by a factor \((\gamma/2)(2\gamma^2) = \gamma^3:\)

\[\nu_s = \gamma^3 \nu_B = \gamma^2 \nu_L = \gamma^2 \frac{eB}{2\pi mc}\]  \(\text{(2.7)}\)

This is the frequency at which the maximum power emission is expected.

The detailed calculation shows that the power emission formula from a single electron in a magnetic field \(B\) is given by [12]:

\[P_e(\nu, \gamma, \theta) = \frac{\sqrt{3}e^3B \sin \theta}{2\pi mc^2} F(\nu/\nu_c)\]

\[F(x) = x \int_x^\infty K_{5/3}(y) dy\]  \(\text{(2.8)}\)

\[\nu_c = \frac{3}{2} \nu_s \sin \theta\]

where \(\theta\) is the angle that the velocity of the electron forms with the magnetic field (pitch angle) and \(K_{5/3}(y)\) is the modified Bessel function of order 5/3.

Assuming an electron density \(n(\gamma, \theta)\), where \(n(\gamma, \theta)d\gamma d\theta\) is the number of electrons per unit volume with Lorentz factor between \(\gamma\) and \(\gamma + d\gamma\) and pitch angle between \(\theta\) and \(\theta + d\theta\), the power emitted per unit volume and unit frequency is:

\[\epsilon_s(\nu) = \int d\gamma \int d\Omega n(\gamma, \theta) P_e(\nu, \gamma, \theta).\]  \(\text{(2.9)}\)

In case of an isotropic emission (like in the case of SNRs), the electron density becomes \(n(\gamma, \theta) = n(\gamma)/4\pi\). Hence, substituting the previous equations and recalling that the particle density is related to the differential flux (number of particles crossing a unit area in a unit time and per unit solid angle) through the relation \(dN/d\gamma = n(\gamma)\beta c/4\pi\):

\[\epsilon_s(\nu) = \frac{\sqrt{3}e^3B}{2\pi mc^2} \int d\gamma \frac{n(\gamma)}{4\pi} \int d\theta \sin^2 \theta F(\nu/\nu_c) = \]

\[= \frac{\sqrt{3}e^3B}{4\pi mc^2} \int d\gamma \frac{4\pi dN}{\beta c} \int d\theta \sin^2 \theta F(\nu/\nu_c) = \]

\[= \frac{\sqrt{3}e^3B}{2\pi mc^2} \int d\gamma \frac{4\pi dN}{\beta c} R(\nu/\nu_c),\]  \(\text{(2.10)}\)
where

$$R(x) = \frac{1}{2} \int d\theta \sin^2 \theta F(x).$$  \hspace{1cm} (2.11)$$

The total emissivity is obtained by integrating over the solid angle, i.e. by multiplying by $4\pi$:

$$Q_\gamma = 4\pi \epsilon_s.$$  \hspace{1cm} (2.12)$$

An important characteristic of the resulting emissivity is that a power law electron distribution produces a power law emissivity spectrum and the two spectral indices are related. If $\alpha$ is the spectral index of the radiation and $s$ the one of the injection, it can be demonstrated (see Appendix A) that

$$\alpha = \frac{s - 1}{2}.$$  \hspace{1cm} (2.13)$$

The synchrotron emission contributes to the radio to X-ray energy range, depending on the maximum of the electron population.

### 2.3.2 Inverse Compton scattering

Inverse Compton scattering consists of the interaction of a high energy electron with a low energy photon; the electron transfers part of its energy to the seed photon, creating a gamma-ray. If $\gamma$ is the Lorentz factor of the relativistic electron ($\gamma \gg 1$) and $\epsilon_0$ is the energy of the seed photon in the laboratory system, the energy of the photon in the rest frame of the electron becomes:

$$\epsilon = \epsilon_0 \gamma (1 + \beta \cos \theta),$$  \hspace{1cm} (2.14)$$

where $\theta$ is the angle between the photon and the electron direction in the laboratory system and $\beta = v/c$.

In scattering off the electron in the rest frame of the electron, the photon goes off at an energy $\epsilon'$ and scattering angle $\Theta'$. The energy $\epsilon'$ after scattering is given by:

$$\epsilon' = \frac{\epsilon}{1 + (\epsilon/mc^2)(1 - \cos \Theta')}.$$  \hspace{1cm} (2.15)$$

\footnote{The notation $Q_\gamma$ in this case is not completely appropriate, since the synchrotron emission does not produce $\gamma$-rays, but we still adopt it in conformity with the other processes.}
Now, in the Thomson limit ($\epsilon \ll mc^2$), $\epsilon' \approx \epsilon$. Going back to the laboratory system an equation similar to (2.14) holds. Then, taking into account that the two angular factors (one per each Lorentz transformation) at most are equal to 2, the maximum photon energy is approximately $\epsilon_1 \approx 4\gamma^2\epsilon_0$. Considering the typical energy of the CMB, the IC contributes mostly to the $\gamma$-ray energy range, with a peaked spectrum characterised by a maximum photon energy depending on the maximum energy of the electrons.

The cross-section of the process can be calculated using Feynman diagrams (see figure 2.4). The result is given by the Klein-Nishina formula \[\text{[12]}\):

\[\sigma_{\text{KN}}(\gamma, \omega_0, \omega) = \frac{2\pi r_0^2}{\omega_0\gamma^2} \left[ 1 + q - 2q^2 + 2q\ln(q) + \frac{\Gamma^2q^2(1-q)}{2(1+\Gamma q)} \right],\]

(2.16)

\[q = \frac{\omega}{4\omega_0\gamma(\gamma - \omega)},\]

(2.17)

where $\Gamma = 4\omega_0\gamma$ and $\omega = h\nu/(mc^2)$; $\omega_0$ is the corresponding quantity for the seed photon.

In the general case, seed photons are not monochromatic, but they are described by a density $n(\omega_0)$. For this reason, the previous cross-section must be integrated over all possible seed photons and all possible electrons, whose density is given by the injection spectrum:

\[Q_\gamma(\omega) = 4\pi \int d\omega_0 \int d\gamma \frac{dN_e(\gamma)}{d\gamma} n(\omega_0)\sigma_{\text{KN}}(\gamma, \omega_0, \omega).\]

(2.18)

\[\text{Figure 2.4: Feynman diagrams for Inverse Compton Scattering.}\]


2.3.3 Bremsstrahlung radiation

When an electron interacts with the Coulomb field generated by charged particles, it radiates photons up to the \( \gamma \)-ray energy band. The process is described in Quantum Electrodynamics by the Feynman diagrams in figure 2.5. Only the relevant diagrams have been considered, which are the ones where a high energy electron radiates a hard photon.

Bremsstrahlung of high energy electrons on target protons, electrons and helium nuclei can be considered. The cross-section of the process is given by the Bethe-Heitler formula for the electron-hadron interaction, while some relativistic corrections are needed for the electron-electron process \[13\]. In this case only the electron-proton contribution is taken into account. The Bethe-Heitler cross-section derived in Born approximation is:

\[
\frac{d\sigma_{e-p}}{d\epsilon} = \frac{4r_0^2\alpha}{\epsilon} \left[ 1 + \left( \frac{\gamma - \epsilon}{\gamma} \right)^2 - \frac{2}{3} \frac{\gamma - \epsilon}{\gamma} \right] \left[ \ln \left( \frac{2\gamma(\gamma - \epsilon)}{\epsilon} \right) - \frac{1}{2} \right],
\]

(2.19)

where \( r_0 \) is the classical radius of the electron, \( \alpha \) is the fine structure constant, \( \gamma \) is the Lorentz factor of the relativistic electron and \( \epsilon \) is the photon energy.

The total photon rate is obtained by integrating the cross-section with the electron injection spectrum. The target density is assumed to be constant, which means that the integration over target density is straightforward.

The final expression for the efficiency is:

\[
Q_{\gamma}(\epsilon) = 4\pi n_H \int d\gamma \frac{dN_e(\gamma)}{d\gamma} \frac{d\sigma_{e-p}}{d\epsilon},
\]

(2.20)

where \( n_H \) is the target proton density, also called ambient density.

Figure 2.5: Feynman diagrams for bremsstrahlung.
2.3.4 Proton-proton interaction

The interaction between high energy protons and target protons is more difficult to describe. This process involves the interactions between quarks, which need Quantum Chromodynamics. A small contribution comes also from the interaction of protons with target nuclei (mostly helium). In proton-proton deep inelastic scattering many particles are produced, among which pions: \( \pi^0 \) decay almost immediately in 2 \( \gamma \)-rays (the mean life of a \( \pi^0 \) is \( 8.5 \cdot 10^{-17} \)s), while charged pions decay in muons and neutrinos (with a mean life of \( 2.6 \cdot 10^{-8} \)s):

\[
p + p \rightarrow \pi + X,
\]

\[
\pi^0 \rightarrow \gamma + \gamma \quad (98.8\%)
\]

\[
\pi^+ \rightarrow \mu^+ + \nu_\mu \quad (99.9\%), \quad \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu
\]

\[
\pi^- \rightarrow \mu^- + \bar{\nu}_\mu \quad (99.9\%), \quad \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu
\]

The pion production from proton-proton inelastic scattering was described through a parametrisation of the cross section in [14]. This first attempt was then improved at momentum below 2 GeV/c in [15], by introducing the contribution of two barionic resonances: \( \Delta(1232) \), corresponding to the physical resonances of mass around 1232 MeV/c\(^2\) and width of approximately 117 MeV/c\(^2\), and \( \text{res}(1600) \), which includes the contribution of different resonances with mass around 1600 MeV/c\(^2\).

In this model, the inclusive cross section for the pion production is divided in three contributions, one coming from the diffractive interaction, another from the non-diffractive interaction and the other from the two resonances. The total p-p cross section evaluated with this model is represented in figure 2.6 and compared to experimental cross sections, which are described by a by-eye-fit curve. In figure 2.7, the total inelastic cross section obtained from the model is compared to the empirical inelastic cross section, defined as the difference between the two by-eye best-fit curves of the experimental total and elastic cross sections.

For each component, the inclusive cross sections for the production of secondary particles (\( \gamma \)-rays, \( e^\pm \) and \( \nu \)) are studied and parametrised independently. First, the secondary spectra were extracted generating events of monoenergetic protons. Then, these spectra were fitted with a common parametrised function. Finally,
the parameters determined for monoenergetic protons are fitted as functions of proton energy, separately for each component. The functional formulae obtained often introduce tails extending beyond the energy-momentum conservation limits. Hence, other functions were introduced to impose manually the kinematic limits. The total cross sections are obtained by summing the three contributions. The event generation was performed using some event simulators, such as Pythia 6.

Once the differential inclusive cross section for the $\gamma$-ray production $d\sigma(E)/d\ln(E)$ was parametrised, the $\gamma$-ray emissivity can be calculated:

$$Q_\gamma(E) = 4\pi n_H \int_{E_{p,th}} dE_p dN_p(E_p) \frac{d\sigma(E)}{dE},$$

(2.21)

where $E_{p,th}(E)$ is the minimum energy of the proton to be able to produce a $\gamma$-ray with energy $E$.

Figure 2.8 shows two examples of $\gamma$-ray spectra produced with this model, taken
Figure 2.7: Empirical inelastic p-p cross sections (small circles), as a function of proton momentum, compared to the total inelastic p-p cross section evaluated with the model described in [15]. Lines are the same as in figure 2.6.

from [15], assuming a simple power-law for the proton spectrum with two different values of the spectral index. It can be noticed that the slope of the $\gamma$-ray spectrum is the same as the proton spectrum. A cut-off for energy below 100 MeV is observed in both cases, due to the threshold energy dictated by the mass of the $\pi^0$.

2.4 Supernova Remnants at high energies

In the previous section, it has been shown that particles accelerated by the SNR shock can produce photons in a wide energy range. A multi-wavelength analysis of the SNR spectrum can give strong constraints on the SNR environment and on the populations of accelerated particles. The accelerated proton spectrum is obviously the most interesting in order to have hints of the acceleration of cosmic rays in SNRs and therefore, the $\gamma$-ray spectrum acquires a key role in the search for evidences of the so called SNR paradigm for CRs. In fact, three of the four previously described processes contribute to the $\gamma$-ray spectrum and usually one of them is expected to be the dominant one, even though models including different contribution are possible. The photon flux in general depends on many parameters characterising the SNR, such as the ambient density or the magnetic field. In
most cases, these parameters are not known, but some constraints are dictated by multi-wavelength observations. Therefore, it might be difficult to discern among the different contributions at high energy.

One of the most convincing way of proving the hadronic origin of the $\gamma$-ray flux is the observation of the shape of the spectrum at energies below 100 MeV, where the $\pi^0$-decay spectrum presents a break due to the threshold energy of the pion production, often called pion bump. In this frame, Fermi-LAT experiment, which is sensitive to an energy range which goes from few tens of MeV to few hundreds of GeV, does have an essential role. Furthermore, a new reprocessing of data
is ongoing within the Fermi-LAT collaboration, which will significantly increase the sensitivity of the instrument below 100 MeV and will probably either provide evidence of the acceleration of CRs or open a new scenario in our knowledge of cosmic ray origin.

### 2.4.1 SNRs with Fermi-LAT

The two most famous claims of cosmic ray acceleration in SNRs are IC 443 and W44 [16], two very bright SNRs in which the characteristic pion-decay feature was detected. Figure 2.9 shows the Fermi-LAT measurements for the two SNR, represented as Spectral Energy Distribution (SED), which is obtained multiplying the differential photon flux by $E^2$, in order to put in evidence the features of the spectrum. Data are compared to different models obtained taking into account the different contributions to the $\gamma$-ray spectrum. It can be noticed that in both cases the data points at energies around and below 100 MeV strongly support the $\pi^0$-decay hypothesis.

However, the shape of the two spectra is a bit more complicated with respect to what the Fermi acceleration theory predicts. As described in [16], the $\gamma$-ray spectrum is well described by a model in which a broken power-law is assumed for accelerated protons, which is not predicted by the Fermi mechanism. The existence of this break can be explained by the fact that these two SNRs are not at the initial state of their evolution, but they are interacting with a dense region of gas called molecular cloud (MC). This is also the reason why these two SNRs are very bright, being the photon flux proportional to the target proton density. Being these SNRs in an advanced stage of evolution, high energy cosmic rays are not confined within the SNR shell any more, resulting in a break of the proton spectrum and hence of the $\gamma$-ray spectrum.

Other important detections of SNRs by the Fermi-LAT experiment are the cases of Cassiopeia A (Cas A) and RX J1713.7-3946, which are two shell-type young SNRs. [17] [18] Figures 2.10 and 2.11 show the SED of the two SNRs including the Fermi-LAT data points and some interpretation models of the $\gamma$-ray spectrum. Being these SNRs very young, they are much less brighter than the two previous cases, since there is no interaction with a molecular cloud. Cas A is another candidate to support the cosmic ray acceleration in SNRs, since Fermi-LAT points
Figure 2.9: Gamma-ray spectra of IC 443 (A) and W44 (B) as measured with the Fermi LAT. Color-shaded areas bound by dashed lines denote the best-fit broadband smooth broken power law from 60 MeV to 2 GeV; gray-shaded bands show systematic errors below 2 GeV due mainly to imperfect modelling of the galactic diffuse emission. At the high-energy end, TeV spectral data points for IC 443 from MAGIC and VERITAS are shown, while magenta stars denote measurements from the AGILE satellite for these two SNRs. Solid lines denote the best-fit pion-decay gamma-ray spectra, dashed lines denote the best-fit bremsstrahlung spectra, and dash-dotted lines denote the best-fit bremsstrahlung spectra when including an ad hoc low-energy break at 300 MeV $c^{-1}$ in the electron spectrum. These fits were done to the Fermi LAT data alone. [16]
Supernova Remnants as Cosmic-ray sources

Figure 2.10: Gamma-ray spectrum of Cas A together with the emission models. The Fermi, MAGIC, and VERITAS points are plotted as filled circles, triangles, and open circles, respectively. The curves show a leptonic model (dashed line) and a hadronic model (solid line). See [17] for more details.

give some hints to prefer the hadronic model to the leptonic one, thanks to the upper limits at low energy.

The case of RXJ17.13.7-3946 is different. As can be seen in figure 2.11 Fermi-LAT data show that the $\gamma$-ray spectrum has a different shape with respect to the previous cases, which can be better explained through a pure leptonic contribution coming from the inverse Compton scattering of accelerated electrons (figure 2.11b). However, as also stated in the Fermi-LAT publication [18], the leptonic model does not necessarily imply that the proton content in this SNR is small, but the absence of $\gamma$-rays from $\pi^0$ decay can be related to a low ambient density, which reduces the hadronic contribution to the photon flux. Anyway, upper limits on the accelerated proton spectrum can be deduced.

Another important case of a shell-type young SNR is Tycho, which originated from a Type Ia supernova explosion in 1572. Tycho represents another example to look for evidence of hadronic interaction in SNRs. For this reason it has been studied in this work and will be discussed in chapter 4.

These few examples suggest that the most favourable scenario to study the proton population is searching for regions of the sky with a high-density gas, in order to have a high photon flux. Figure 2.12 shows two possible way in which accelerated cosmic rays can interact with a molecular cloud. In the first case the shock of the SNR crushes with the MC and a re-acceleration of CRs takes place. In the second
Figure 2.11: Gamma-ray spectrum of RX J1713.7-3946 as measured by Fermi-LAT and HESS experiments. The green and grey bands include the statistical and systematic uncertainties for the Fermi-LAT data. The curves represent a range of proposed models to account for the $\gamma$-ray spectrum. These models have been generated previously to the LAT detection and hence match the TeV emission only. In fig. (A) $\pi^0$ dominant models are shown, while in fig. (B) leptonic models are shown. See [18] for more details.
case, CRs escape from the acceleration region and illuminate the MC. In both cases, the observed $\gamma$-ray emission must necessarily take place well after the SN explosion, since either the shock or the CRs must reach the MC. However, as it has been noticed in the two cases of interacting SNR described, the spectrum observed presents features which are not directly reproducible with the pure acceleration theory, but other phenomena related to the SNR environment and evolution must be taken into account. Furthermore, great care is needed in order to correctly disentangling the source and the illuminated cloud both for the spatial extension and for the spectral emission.

On the other hand, young SNRs, being at the initial state of their evolution, have much simpler environments. Even though in most cases, due to the low ambient density, they are less bright in the MeV-TeV energy range, direct observations made at all wavelengths may give very detailed information about the shock generated by the SN explosion and about its propagation in the interstellar medium. Furthermore, a detection in the $\gamma$-ray energy range could give clear information about the acceleration taking place in the SNR. Hence, these cases represent the ideal situations to study and test the acceleration theory, since no many other factors are to be taken into account.

In this sense, a more systematic study of SNRs in the $\gamma$-ray energy range, which is already ongoing within the Fermi-LAT collaboration with the production of the first SNR Fermi catalog, is necessary to obtain more complete information about the origin of cosmic rays and confirm or contradict the SNR paradigm of cosmic rays.
Chapter 3

\(\gamma\)-ray data analysis with Fermi-LAT

3.1 Fermi-LAT experiment

The Large Area Telescope (LAT) is the primary instrument on board the Fermi Gamma-ray Space Telescope (Fermi)\(^1\), launched on June 11, 2008. The LAT is an imaging telescope detecting photons from 20 MeV to more than 300 GeV. The second instrument on board Fermi is the \(\gamma\)-ray burst monitor (GBM), dedicated to the study of transient phenomena in the 8 keV - 40 MeV energy range, ensuring a good overlapping with the LAT sensitivity.

3.1.1 The LAT on orbit

Fermi is on orbit around the Earth at an altitude of \(\approx 565\) km with an inclination of \(25.6^\circ\) with respect to the Equator. The Fermi observatory started the Science operations on August 13, 2008. The primary observing mode for Fermi is a scanning mode, where the LAT observes the whole sky every two orbits in approximately 3 hours. Fermi is occasionally operated in pointing mode to study interesting transient phenomena. Calibration runs are also periodically performed.

\(^1\)http://fermi.gsfc.nasa.gov/; formerly known as Gamma-ray Large Area Space Telescope (GLAST)
The LAT spends a fraction of its orbit in the South Atlantic Anomaly (SAA), which is an area in the south hemisphere where the Van Allen radiation belt comes close to the Earth’s surface (at an altitude of about 200 km). This leads to an increased flux of energetic particles, leading to the saturation of the instrument electronics, exceeding the safe operation limits and producing a rapid deterioration of the detector. For this reason, the LAT does not operate when crossing the SAA. The perimeter delimiting the SAA was conservatively defined prior to launch. As a result, the exposure time in the south hemisphere is reduced with respect to the northern one and a fraction of about 13% of the total observation time is lost.

### 3.1.2 The LAT detector

The LAT is an imaging, wide-field-of-view, $\gamma$-ray telescope, detecting photons from 20 MeV to more than 300 GeV. The LAT is a pair-tracking telescope, made from a $4 \times 4$ array of towers, with a converter-tracker and a calorimeter module. A segmented anticoincidence detector (ACD) covers the tracker array and a programmable trigger and the data acquisition system (DAQ) complete the instrument. Figure 3.1 provides a schematic illustration of the LAT.

High-energy $\gamma$-rays interact with matter mainly through production of $e^+e^-$ pairs. The LAT converter-tracker modules are made of 16 planes of high-Z material.
(tungsten) in order to promote the conversion of $\gamma$-rays into pairs, interleaved with 18 planes of silicon strip converters which reveal the charged particle trajectories. The last two silicon planes have no converter foils in order to accurately measure the entering point in the calorimeter. The thickness of the tungsten layers was optimized to maximize the conversion probability and minimise multiple scattering and bremsstrahlung energy losses of electrons and positrons. The first 12 layers have a thickness of 0.03 radiation lengths to maximize the angular resolution at low energies limiting the Coulomb scattering (front section), while the last 4 layers have a thickness of 0.18 radiation lengths to maximize the conversion probability at high energies (back section). In order to obtain a two-dimensional reconstruction of the particle positions, each silicon plane is made of two layers ($x$ and $y$) of single-sided silicon strip detectors. The aspect ratio of the tracker (height/width) is 0.4, allowing a large field of view (FoV) of 2.4 sr and ensuring that nearly all pair-conversion events will pass into the calorimeter. The choice of silicon-strip detectors gives to the tracker a self-triggering capability, getting rid of the necessity of an external trigger. In addition, all of the LAT instrument subsystems utilise technologies that do not use consumables, such as the gas of tracking spark chambers used in previous high-energy $\gamma$-ray telescopes.

Every calorimeter module consists of 8 layers of 12 CsI crystals activated with thallium each, with a total depth of $\approx 8.6$ radiation lengths, giving both longitudinal and transverse information about the energy deposition pattern. Each crystal is read out by two photodiodes, one at each side, providing three spatial coordinates: two discrete coordinates from the location of the crystal in the array and the third coordinate given by measuring the light yield asymmetry at the ends of the crystal along its longitudinal dimension.

The calorimeter provides the measurement of the energy deposited by the shower initiated by the $e^+e^-$ pairs, allowing the energy reconstruction of the incoming photon. Furthermore, it images the shower development, enabling the estimation of the energy leakage and the discrimination from hadronic showers.

The Anti-coincidence Detector (ACD) allows the discrimination of photons from the large background given by charged cosmic rays. It consists of plastic scintillators hermetically enclosing the tracker and the calorimeter, which respond only to the passage of charged particles. However, it is essential to avoid the “self-veto” effect, i.e. the rejection of $\gamma$-rays with energies above 10 GeV because of the back-splash on the ACD of secondary particles produced in the calorimeter. For this
purpose the ACD is segmented into 89 plastic scintillator tiles providing spatial information that can be correlated with the signal from tracker and calorimeter modules. Scintillation light from each tile is recorded by wavelength shifting fibers embedded in the scintillator and connected at their ends to two photomultiplier tubes (PMTs). A complete description of the

An important remark is that the ACD system permits also the detection of charged particles, in particular electrons and positrons, allowing high precision measurements of the electron plus positron spectrum (see figure 1.6). In this case, obviously, the scintillators must be used in coincidence with the tracker signals. Since Fermi-LAT experiment was not designed for the detection of cosmic rays, it does not have a magnetic spectrometer and cannot directly measure the charge of the particles. However, techniques based on the geomagnetic field have been developed to distinguish positrons and electrons (see [21]) in order to measure charge dependent quantities such as the positron fraction.

### 3.1.3 Data acquisition

The DAQ collects information from the subsystems, generate the instrument triggers (at a rate of 2-3 kHz) and provides the first onboard filter reducing the rate of downlinked events to approximately 500 Hz, while the entire LAT instrument is connected to the satellite through the spacecraft interface unit (SIU), which contains the control system of the satellite, collects data from the different towers and sends them to the Solid State Recorder (SSR) and then to the Earth.

The tracker and calorimeter modules of each tower are interfaced by a tower electronics module (TEM), generating tower-based triggers. At the entire instrument level a global unit collects signals from all the electronics modules, provides an interface with the ACD and generates a global trigger based on the information received from the TEMs and the ACD. After triggering and building the events with the information received from the whole apparatus, it sends them to the event processor units (EPUs).

The minimum read-out time per event is 26.5 $\mu$s, due to the transmission of the trigger signal between the different units. During the event read-out the different subsystems send a busy signal to the global unit, which generates the overall dead time. The trigger is designed in order to minimise the dead time due to
background events. Triggers are generated by any of the TEMs, either if there is a coincidence of at least three silicon planes in a row or the energy deposit in any of the calorimeter crystals exceeds a fixed threshold. The events triggering the LAT are indeed mostly background due to CR interactions. The two EPUs implement the onboard filtering aimed at reducing the contamination by charged particles and maximising the efficiency for γ-ray detection.

### 3.1.4 Event classification

Event classification aims at selecting the best estimates of the event direction and energy and determining their accuracy, as well as at drastically reducing the backgrounds in the final data sample. The background rejection is a fundamental task since the background events triggering the LAT exceed by $10^5$ the celestial γ-ray flux.

The various scientific objectives of the LAT require an appropriate tuning of the instrument performance and of the efficiency and background contamination; a few event classes were defined prior to launch. The Transient class, suitable for studying localised, intense, transient phenomena, has the largest efficiency but a residual background at the level of the γ-ray detection rate. The Source class has a better background rejection than the Transient class and is primarily intended for point-source analysis and diffuse analysis at galactic latitudes lower than $30^\circ$. The Clean class is identical to the Source below 10 GeV. Above 10 GeV this selection has a significantly lower background rate than the Source. This class is intended for diffuse analysis at high galactic latitudes ($>30^\circ$). The Ultraclean class is the purest class and is primarily intended for diffuse analyses which require a very low background contamination.

All the event classes have a residual background contamination. The reducible backgrounds are given by events which are erroneously classified as γ-rays and, in principle, could be identified as background and eliminated. The irreducible backgrounds are given by charged particles interacting with the dead materials surrounding the instrument and producing a real γ-ray which is then detected by the LAT. Irreducible events constitute the majority of residual backgrounds in the purest classes and cannot be completely eliminated.
After the event is analysed, a list of predefined quantities, such as energy and arrival direction, is associated with the photon event, as well as a flag indicating the event class. A full list of these quantities is shown in figure 3.2. LAT photon data are publicly available through the Fermi Science Support Center (FSSC [22]), together with the orbital history of the telescope.

3.1.5 Instrument Response Functions (IRFs)

Data analysis cannot be performed without a detailed description of the instrument performances, which is fulfilled through the Instrument Response Functions (IRFs).

The LAT IRFs were defined and parametrised prior to launch based on the MC simulations (Pass 6 or P6 IRFs). After launch the IRFs started being modified to take into account effects observed on-orbit (P7 and P7 reprocessed IRFs). The LAT IRF models developed by the LAT team are released along with data. More details on the performances of the different releases can be found on the Fermi Science Support Center [22].

A re-processing of LAT data is ongoing to develop a new set of IRFs, the Pass 8 or P8 IRFs, which will be released presumably next year along with the reprocessed data. In this reprocessing, thanks to the detailed comprehension of the instrument acquired during the 6-year activity of the LAT, the event reconstruction technique
is being renewed, in order to be able to trigger the events in which more than one \( \gamma \)-ray impinge on the detector. Figure 3.3 gives a schematic representation of this new technique. Figure 3.4 shows the expected increase in the acceptance of the instrument with respect to the previous P7 reconstruction technique, which is especially significant at low energies, where the probability of a multiple-photon event is higher, given the higher flux of \( \gamma \)-rays.

**Figure 3.3:** Schematic representation of the Pass 8 reconstruction technique. [23]

**Figure 3.4:** Expected acceptance of the instrument with the Pass 8 event reconstruction, compared to the latest P7 Reprocessed performance. [23]

**IRF definition**
The IRFs are defined as a function \( R \) of the true photon energy \( E' \) and direction \( \hat{p}' \), the measured photon energy \( E \) and direction \( \hat{p} \) and time \( t \), so that the differential count rate measured by the instrument is given by the convolution of the true
differential flux per unit area at the detector with the IRFs:

$$\frac{dN}{dt dE d\hat{p}}(E, \hat{p}, t) = \int dE' d\hat{p}' R(E, \hat{p}|E', \hat{p}'; t) \frac{dN}{dt dE d\hat{p}' dS}(E', \hat{p}', t). \quad (3.1)$$

The scaling factor $T(t)$ accounts for temporal variations, such as instrument failures or the deterioration of instrument components. The lack of consumables makes the LAT performance very stable and therefore this term is negligible. $R$ is then defined as the product of three independent factors:

- the effective area, $A(E', \hat{p}')$, which is the detection efficiency for photons of true energy $E'$ and arrival direction $\hat{p}'$ expressed as an area;

- the Point Spread Function (PSF), $P(\hat{p}|E', \hat{p}')$, which is the probability density that a photon with energy $E'$ and arrival direction $\hat{p}'$ has a reconstructed direction $\hat{p}$;

- the energy dispersion, $D(E|E', \hat{p}')$, which is the probability density that a photon with energy $E'$ and arrival direction $\hat{p}'$ has a reconstructed energy $E$.

Figures from 3.5 to 3.8 show some plots summarising the LAT performance for the latest version of the Pass 7 reprocessed IRFs for the \textit{SOURCE} event class, the P7REP\_SOURCE\_V15, which is the latest version of IRFs publicly available on the FSSC. Figure 3.5 shows the effective area as a function of energy for normally incident photons and as a function of incidence angle for 1 GeV photons. The relevant quantity is the effective area integrated over the FoV, called acceptance. Figure 3.6 shows the intrinsic acceptance, regardless of orbital characteristics. Since the effective area depends on the so called \textit{off-axis angle}, which is the angle between the direction of the incoming photon and the $z$-axis of the instrument frame of reference, the true acceptance is obtained weighting the effective area with the time fraction that the LAT spends observing the incoming photon direction at a certain off-axis angle. This calculation is performed directly during the analysis procedure, since depends on the region of the sky observed and on the spacecraft history.

Figure 3.7 shows the PSF for 68\% and 95\% event containments as a function of energy. The PSF strongly depends on energy, being much larger at low energy where the Coulomb scattering in the tracker affects significantly the particle trajectories.
Figure 3.5: The LAT effective area for the P7REP_SOURCE_V15 IRF as a function of energy for normally incident photons (A) and as a function of incidence angle for 1 GeV photons (B). The curves correspond to front-converting events (red), back-converting events (blue) and total (black).

Figure 3.6: The LAT acceptance for the P7REP_SOURCE_V15 IRF as a function of energy. The curves correspond to front-converting events (red), back-converting events (blue) and total (black).

The PSF is a peaked distribution around the true position, as one would expect, but it has larger tails with respect to an ideal Gaussian distribution, especially at energies above 10 GeV.

Finally, figure 3.8 shows $\Delta E/E$ for 68% and 95% event containment, which is a measure of the instrument energy resolution. It is better than 15% over most of the LAT energy band.
3.2 Likelihood analysis

Once data have been collected, a detailed analysis of their spatial and spectral distribution must be performed in order to obtain information of astrophysical interest. However, in most cases it is not possible to isolate a specific source in high-energy \(\gamma\)-rays, because of the presence of a bright and structured background given
by interstellar emission. Furthermore, the angular resolution, strongly varying
with energy, is often poor compared to other wavelengths and in most cases is not
sufficient to observe the extension of the source, which then appears as a point
source. Therefore, statistical techniques have to be applied to study the GeV sky.

The method adopted is the likelihood maximisation, based on the Poisson statistics
used to describe the photon count distributions.

Let \( M(E', \hat{p}', t; \alpha_k) \) be the differential flux per unit area describing the observed
region of the sky as a function of the true energy of the photons \( E' \), their true
arrival direction \( \hat{p}' \) and the time of observation \( t \). \( \{\alpha_k\}_{k=1,...,m} \) are the parameters
describing the model, whose best-fit values are determined through the likelihood
maximisation. When steady sources are considered, \( M \) can be considered constant
in time. In this thesis, no variable sources will be taken into account, so the time
dependence of the source spectrum will be neglected.

Using the IRF definition in equation (3.1) and neglecting the time variations of
the IRFs, the observed count rate \( J \) can be derived:

\[
J(E, \hat{p}; \alpha_k) = \int dE' d\hat{p}' R(E, \hat{p}|E', \hat{p}') M(E', \hat{p}'; \alpha_k),
\]
which is a function of the reconstructed energy and arrival direction of the photon.

The total number of counts expected in a solid angle \( \Omega \) and in an energy range
\((E_1, E_2)\) and in a time of observation \((t_1, t_2)\), is obtained by integration over \( E, \hat{p} \)
and \( t \):

\[
\Lambda(\alpha_k) = \int_{t_1}^{t_2} dt \int_{E_1}^{E_2} dE \int_{\Omega} d\hat{p} J(E, \hat{p}; \alpha_k).
\]

Expected counts can be compared with observed counts through the Poisson dis-
tribution

\[
P(n; \lambda) = \frac{\lambda^n}{n!} \exp(-\lambda).
\]

Given a set of values of \( \{\alpha_k\}_{k=1,...,m} \), \( \Lambda(\alpha_k) \) represents the true value of counts, i.e.
the expectation value of the Poisson distribution. Therefore, \( P(N_{\text{obs}}; \Lambda(\alpha_k)) \) is the
probability of observing \( N_{\text{obs}} \) photons given the model parameters \( \{\alpha_k\}_{k=1,...,m} \).
\( P(N_{\text{obs}}; \Lambda(\alpha_k)) \) has to be maximised with respect to the model parameters, in
order to find the best estimates for \( \{\alpha_k\}_{k=1,...,m} \).
The analysis is usually performed dividing the observed region of the sky in pixels and in energy bins (binned analysis). If the statistic is low or the pixelisation of the sky is sufficiently fine so that each pixel contains either 0 or 1 photons, an unbinned analysis is performed, in which each single photon contributes directly to the likelihood. The likelihood is defined as the product of the probabilities of each pixel \( i \) and each energy bin \( j \):

\[
\mathcal{L} = \prod_{i,j} P(N_{i,j}; \Lambda_{i,j}(\alpha_k)). \tag{3.5}
\]

It is convenient to consider the natural logarithm of the likelihood:

\[
\ln \mathcal{L} = \sum_{i,j} N_{i,j} \ln \Lambda_{i,j}(\alpha_k) - \sum_{i,j} \Lambda_{i,j}(\alpha_k) - \sum_{i,j} \ln(N_{i,j}!) \tag{3.6}
\]

Since the last term does not depend on the model parameters, it can be eliminated and the log-likelihood becomes

\[
\ln \mathcal{L} = \sum_{i,j} N_{i,j} \ln \Lambda_{i,j}(\alpha_k) - \sum_{i,j} \Lambda_{i,j}(\alpha_k) = \sum_{i,j} N_{i,j} \ln \Lambda_{i,j}(\alpha_k) - \Lambda_{\text{tot}}(\alpha_k), \tag{3.7}
\]

where \( \Lambda_{\text{tot}}(\alpha_k) \) is the expected total counts.

The likelihood profile around the maximum gives information about the errors on the best-fit values. If the statistics is sufficiently high, the log-likelihood becomes a parabola near the maximum (and the likelihood becomes a Gaussian distribution). The statistical error on the parameters is defined as the variations of the parameters that causes a drop of the log-likelihood maximum value of 0.5:

\[
\ln \mathcal{L}(\alpha_k \pm \sigma_{\alpha_k}) = \ln \mathcal{L}(\bar{\alpha}_k) - 0.5, \tag{3.8}
\]

where \( \bar{\alpha}_k \) is the best-fit value that maximise the log-likelihood.

The quantitative comparison between different models using the likelihood analysis can be performed using the likelihood ratio test (LRT). Let \( M_0(\{\alpha_k\}_{k=1,..,h}) \) be a simpler model than the one considered before, that means \( h < m \). For example, \( M_0 \) can be obtained fixing the values of \( m - h \) parameters of \( M \). The test statistics is defined as

\[
TS = 2(\ln \mathcal{L} - \ln \mathcal{L}_0), \tag{3.9}
\]
where $\ln \mathcal{L}$ and $\ln \mathcal{L}_0$ are the maximum values of the log-likelihood found using the model $M$ and $M_0$, respectively. The Wilk’s theorem [24] states that the TS-value is distributed asymptotically as a $\chi^2$ with $m - h$ degrees of freedom. When the TS is large the $M_0$ hypothesis is rejected and the complete model $M$ is adopted. The confidence level at which the full model $M$ describes data better than the simpler model $M_0$ is

$$c.l. = \int_0^{TS} \chi^2_{m-h}(s) ds,$$

(3.10)

being $\int_{TS}^{+\infty} \chi^2_{m-h}(s) ds$ the chance probability that the test statistics is larger than the obtained value.

### 3.3 Fermi Science Tools

The LAT collaboration developed a set of tools, called the Science Tools, to perform data analysis at high level. They are publicly available through the FSSC [25]. The Science Tools include tools to rapidly simulate LAT observations, to perform temporal analysis of pulsars, select and explore LAT data and perform the likelihood analysis described in section 3.2. In this section a brief descriptions of the tools used to perform the analysis developed in this thesis will be provided.

#### 3.3.1 Data selection

The online data analysis, made on board by the LAT, produces tables of events containing a list of predefined quantities per each event (see figure 3.2). The data selection tools allow the user to select events according to one or more of these quantities. For example, photons from a specific region of the sky, in a specific energy range or time interval can be selected. As already discussed above, events are classified in different classes, according to the degree of background contamination, so an event-class selection must be performed.

Another type of selection regards the contamination coming from $\gamma$-rays produced in the interaction of cosmic rays with the Earth’s atmosphere. This background is very bright, but such photons can be easily ruled out thanks to their arrival direction. For this purpose a cut on the zenith angle, which is the angle between the incoming photon direction and the direction perpendicular to the Earth’s surface,
is applied, eliminating all the events which are not within a maximum zenith angle. The tool which is used to perform these cuts on data is the \textit{gtselect} tool.

Another important tool for data selection is \textit{gtmktime}. It performs the calculation of the so called \textit{good time intervals} (GTIs), which are the time ranges when the LAT was actually able to collect data. Only data taken in these GTIs can be considered valid. GTIs are used by other tools to correctly calculate the exposure of the LAT. The \textit{gtmktime} tool performs cuts based on the spacecraft position, for example selecting times when the spacecraft is not in the Southern Atlantic Anomaly (SAA). For this purpose, it needs the spacecraft file, containing all the information about the LAT position and orientation as a function of time.

### 3.3.2 Fitting procedure

The likelihood analysis is implemented through a series of subsequent steps.

The first step consists of the evaluation of the livetime of the LAT, which is the time that the detector spent observing each position of the sky. The tool that performs this calculation is the \textit{gtltcube}, which calculates the livetime as a function of the sky direction and of the angle between this direction and the instrument z-axis, also called off-axis angle. The off-axis angle dependence is important since the IRFs depend on this angle.

The next step is the evaluation of the exposure for the region of the sky under investigation, which has to be divided in pixels of arbitrary dimensions. The exposure is obtained multiplying the livetime calculated using \textit{gtltcube} by the effective area of the instrument parametrised in the IRFs. Since the effective area depends on the off-axis angle, the exposure is obtained weighting the effective area with the livetime calculated at each value of the off-axis angle:

\[
\epsilon_{\text{pix}}(E) = \int l_{\text{pix}}(\theta) A_{\text{eff}}(E) d\theta,
\]

where \(\theta\) is the off-axis angle.

The last step before the fit is the evaluation of the expected and observed counts for each pixel and energy bin. The observed counts are evaluated using the \textit{gtbin} tools, which creates a 2-D map (two spatial coordinates) and a 3-D map (two spatial coordinates plus one dimension for the energy binning) of the observed
photons for the unbinned and binned analysis respectively. The expected counts, according to equation (3.2) and (3.3), are evaluated through the convolution of the model flux of the region of the sky analysed with the exposure previously calculated. This product must also be multiplied by the PSF, which is important for point sources and at low energies. For each pixel and energy bins:

\[ \Lambda_{\text{pix}}(E) = PSF(E)\epsilon_{\text{pix}}(E)M_{\text{pix}}(E), \] (3.12)

where \( M_{\text{pix}}(E) \) represents the model flux in one specific pixel and at a specific value of energy (which theoretically must be calculated by integration over the pixel and energy bin of the model function). In the convolution with the IRFs the energy dispersion has been neglected to limit the computing time, since its contribution is not very important. This can be a source of systematic effects, especially at low energies.

The tool that performs this calculation is gtsrcmaps. The model is defined through a file in the “XML” format, which contains the spectral and spatial shape of each source in the model. The spectral shapes can be chosen among a set of available functions, such as simple power-law, broken power-law, log-parabola\(^2\), power-law with exponential cut-off. The tool gtsrcmaps performs the convolution independently for each of the source in the model file. The different contributions will then be added together during the fit.

Finally, the likelihood fit is performed by the tool gtlike, which defines the log-likelihood using the observed and model counts and finds the maximum using some minimisation algorithms, such as Minuit \([26]\). The tool gtlike returns as output the best-fit values of the model parameters and their statistical uncertainties derived from the likelihood profile, as well as the maximum likelihood logarithm values associated to each model to evaluate the TS value, as described in section 3.2. The TS of the fit is evaluated with respect to a null-hypothesis in which the main source is eliminated from the model.

Finally, the results of the fit can be compared to the data by generating a counts map using the best-fit parameters. This is performed through the tool gtmodel.

\(^2\)Log-parabola is a functional form introduced to curved spectra which cannot be reproduced with an exponential cut-off. The functional form is given by:

\[ N(E) = N_0 (E/E_b)^{\alpha+\beta \ln(E/E_b)}, \] (3.13)

where \( \alpha \) is the spectral index, \( \beta \) is the index of the curvature and \( E_b \) is the energy break.
Another useful instrument to check the goodness of the fit is the TS map. This map is obtained by adding a point-like source to the model and performing a likelihood fit for each position of this test source in the map. The TS value of each fit is then calculated and the map is created. A bright spot in the TS map suggests the presence of a significant excess in the data with respect to the model. The TS map is usually created using another tool, called pointlike (see [27] [28]), which makes some simplifications on the likelihood and allows a much faster maximisation of the likelihood.

### 3.3.3 Source localisation and extended sources analysis

An important aspect of the study of a source is its spatial shape. This problem is complicated by the PSF, which transforms point-like source in extended sources. Therefore, a source cannot be considered extended if the observed dimensions are comparable or lower than the PSF of the instrument. The sources that are sufficiently extended to be spatially resolved are called diffuse sources and require a specific study for their extension.

\( \text{Gtlike} \) can only fit the spectral parameters of the model. An iterative procedure can be used to study the extension of the source, by performing independent fits using different fixed spatial models. The likelihood profile obtained has been shown to correctly reproduce the extension of simulated extended sources assuming that the true position is known. [29]

However, this method requires a high computing time. Furthermore, in order to maximise the statistical significance of the detection, the position, the extension and the spectrum of the source should be simultaneously fit. The pointlike tool allows this simultaneous fit keeping the computing time reasonable. The assumption at the base of the likelihood definition in pointlike is that the source model can be factorised: \( M(x, y, E) = S(x, y) \times F(E) \), where \( S(x, y) \) is the spatial distribution and \( F(E) \) the spectral distribution. The main advantage in pointlike is that the spatial binning of data scales with the PSF, i.e. a coarse binning is adopted where the PSF is large. This technique allows better performances at low energy, where the PSF is large, keeping a good accuracy at high energy. The approximations are performed maintaining an accuracy level of 1%. 
Pointlike is often preferred to localise and find the extension of the source, before
performing a more accurate fit of the spectrum using gtlike. This is the typical
procedure adopted in the development of source catalogs (see [30] and [31]).
Chapter 4

Evidence of hadronic interaction with Fermi-LAT data: the Tycho SNR case

4.1 Introduction

As already discussed in chapter 2, LAT data are an important way to obtain information about the origin of $\gamma$-rays in SNRs. In this thesis I focused my work on Tycho Supernova Remnant, which is a candidate to be source of accelerated protons. The source has already been analysed with 3 years of Fermi data [32], giving some hints of the presence of hadronic interaction. Here, the analysis is performed with the full set of data available and with the new reprocessed Pass 8 data, in order to extend the analysis at low energy and confirm the previous hypothesis. In the analysis, the energy spectrum of the source is obtained and then compared to the data collected by other experiments at different energies. Finally, an interpretation model of the multi-wavelength spectrum is developed.

4.2 Data analysis and results

The analysis procedure is based on the maximum likelihood method described in chapter 3 and it was performed using the Fermi science tools.
The analysis was performed using two sets of data. The first set consists of the Pass 7 reprocessed (P7REP) data, which are the latest data publicly available of the FSSC. The second set of data consists of the new Pass 8 (P8) data (see section 3.1.5 for more details about this data reprocessing), which are being developed within the Fermi-LAT collaboration and will be presumably released next year. The results obtained with this set of data must still be considered preliminary. The comparison of the two analysis allows a very interesting comparison between the Pass 7 and Pass 8 data, highlighting the differences and the improved performances of the Pass 8 data. This work has also been performed in the process of validation of the new Pass 8 data within the collaboration and is currently under development.

Data selection

The data selection was performed using the science tools gtselect and gtmktime, as described in section 3.3.

The region of the sky selected, called region of interest (ROI), is a squared region inscribed in a circle centred on the presumed Tycho position and with a radius of 15°. Even though the source analysed is expected to be point-like, the selection of such a big ROI is necessary due to the PSF of the instrument, which is large at low energies. In fact, the fit must take into account all contributions of the nearby sources. The position of Tycho was initially chosen according to the measurements at other wavelengths, above all in the radio and X-ray bands. A fit of the position was then performed using the tool pointlike, as described in section 3.3.3 and the new coordinates were assumed according to these results.

The time window of data selected is of 60 months for the P7REP data and 58 months for P8 data, which was the largest time window available for P8 data at the time of the analysis. The analysis was performed in the energy range from 60 MeV to 300 GeV, obtaining an important improvement with respect to the results previously published in 32. The details of the selections adopted for the two analysis are summarised in table 4.1. Figure 4.1 represents a counts map summed over energy obtained with Pass 8 data. It can be noticed that the region is dominated by the diffuse emission of the Galactic plane. Some very bright
Chapter 4. Evidence of hadronic interaction with Fermi-LAT data

<table>
<thead>
<tr>
<th>PASS 7 REPROCESSED</th>
<th>PASS 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>P202 data files</td>
<td>P300X data files</td>
</tr>
<tr>
<td>ROI=15°</td>
<td>ROI=15°</td>
</tr>
<tr>
<td>6 Zenith angle &lt; 100°</td>
<td>Zenith angle &lt; 100°</td>
</tr>
<tr>
<td>(t_{\text{start}} = 239557418) MET (04/08/2008)</td>
<td>(t_{\text{start}} = 239557418) MET (04/08/2008)</td>
</tr>
<tr>
<td>(t_{\text{stop}} = 397323820) MET (04/08/2013)</td>
<td>(t_{\text{stop}} = 392846505) MET (13/06/2013)</td>
</tr>
<tr>
<td>Energy range: 60 MeV - 300 GeV</td>
<td>Energy range: 60 MeV - 300 GeV</td>
</tr>
<tr>
<td>Event class: SOURCE</td>
<td>Event class: SOURCE</td>
</tr>
<tr>
<td>IRFs: P7REP_SOURCE_V15</td>
<td>IRFs: P8SOURCE_V20R9P0_V0</td>
</tr>
</tbody>
</table>

Table 4.1: Data selection for the analysis with Pass 7 reprocessed and Pass 8 data.

\(a\) MET = Mission Elapsed Time

...sources emerge over the diffuse background, such as the pulsar in the CTA1 SNR, which is in the upper part of the ROI and is one of the brightest point sources in the \(\gamma\)-ray sky.

Despite the high background, the precise knowledge of the diffuse emission allows the detection of not so bright point sources, such as Tycho. In the P8 data set, there are approximately 3 millions of photons in the ROI surviving to the cuts (2 millions in the P7REP data set). Of these photons, the fit procedure will attribute approximately 1500 to Tycho (1000 with the P7REP analysis), which is 0.05% of the total number of photons.

Analysis method

The analysis chain was developed following the steps described in section 3.2, using the Fermi science tools. Different analysis were performed in order to study the spectrum of the source.

In order to perform the fit, a model was developed for the ROI. Three diffuse sources were included in the model. The first one models the Galactic diffuse background due to the interaction of cosmic rays with the interstellar medium, while the second accounts for an isotropic extragalactic background due to distant sources which are below detection threshold or nearby galaxies such as the Magellanic clouds. The third one is a model for the Earth Limb, which comes from the interaction of cosmic rays with the Earth’s atmosphere. Most of these photons, as already discussed in section 3.3.1, can be eliminated thanks to the cut on the
zenith angle, but due to the high statistics involved, including this emission in the model is recommended. The diffuse models are different for the Pass 7 reprocessed and the Pass 8 analysis, since they are obtained from different sets of data. In both cases the latest available versions within the collaboration were used. In the case of Pass 8 analysis, it must be noticed that these models are still preliminary.

In addition to the diffuse sources, all the already known point-like sources are added to the model. These sources are taken from the third Fermi Catalog (3FGL), which has not been published yet, but is in the final stage of its development within the Fermi-LAT collaboration. This catalog is being developed with the same procedure as the first two Fermi catalogs \cite{3FGL1, 3FGL2} using 4 years of Pass 7 reprocessed data and it will contain more than 3000 sources. In the fit, for all the point sources it was adopted the energy spectrum of the catalog. Since the values of the best-fit parameters in the catalog are obtained with different assumptions with respect to this analysis (for example different diffuse models or different time window of data), the normalisations of all the point source spectra within 7° from the ROI center are fitted, in order to balance the possible discrepancies. All the details about the models adopted in the two analysis are summarised in table 4.2.

The fit was first performed on the full energy range (60 MeV - 300 GeV), in order
Table 4.2: Spectral and spatial models adopted in the P7REP (top) and P8 (bottom) analysis.

to validate the global detection of the source. The spectral model of the source must be chosen among the ones available in the science tools. In this case a simple power-law was adopted, even though it does not fully reproduce the spectral shape of the source, as it will be shown in next section.

Secondly, the energy range was divided in 8 energy bins (2 bins per decade) and an independent fit was performed in each energy bin. The spectral shape is the same as before, but since the energy bins are sufficiently small, only the integral flux in each bin is relevant. If \( F(E) \) is the flux shape adopted in the bin \((E_1, E_2)\), the differential spectrum at the energy \( E_0 = (E_1 + E_2)/2 \) is

\[
\frac{dN}{dE}_{E=E_0} = \frac{N(E)}{\Delta E} = \frac{1}{\Delta E} \int_{E_1}^{E_2} F(E) dE,
\]

where \( \Delta E = E_2 - E_1 \) is the bin width. With this method, which in the following sections will be referred as binned analysis, it is possible to study the actual spectral shape of the source.

It must be noticed that this is a limitation to the fit due to the structure of the science tools, since they do not allow the usage of spectral shapes other than the built-in ones. A possible improvement of these tools which is under development
is the customization of the fitting tools in order to include an arbitrary spectral shape to reproduce the source flux.

An estimate of the systematics uncertainties was performed by alternatively fixing or fitting the diffuse components. Since Tycho is near to the galactic plane, the major contribution comes from the Galactic component, which is then left free in all the fits. The other two components (extragalactic and Earth’s limb) are alternatively fixed or fit. In addition, other fits were performed in which the Galactic diffuse was decreased and increased of the 6% of the best-fit value. The variations in the flux of the source give the estimate of the systematic uncertainties.

From the best-fit results, a map of model counts describing the ROI was created and compared with the observed counts, creating a residual map expressed in units of sigma:

\[ R = \frac{C - M}{M}, \]

where \( R \) is the residual, \( C \) is the observed count and \( M \) is the model count.

Finally, a TS map was created using the pointlike tool, as described in section 3.3.2, in order to put in evidence the regions of the ROI which are poorly described by the model, if any, and therefore confirm the goodness of the fit.

Results

The procedure described in the previous section was applied for the analysis of the \( \gamma \)-ray spectrum of Tycho SNR. Since Tycho is one of the sources which will be included in the 3FGL, the initial position of the source was based on the coordinates of the catalog. The first step was to recalculate the position of the source using pointlike (see section 3.3.3). The position found using the P7REP data was not significantly different from the one of the catalog, since the data sets are the same. On the contrary, the P8 data analysis resulted a significant change in the position, as it will be shown later. The new coordinates found with this method were used as center of the ROI and as the position for the source model.

In table 4.3 the results of the fit on the full energy range are shown for both analysis. The spectral shape adopted to describe the source is a simple power-law.
expressed by the following formula:

\[ \frac{dN}{dE} = N(\gamma + 1) \frac{E^\gamma}{E_{\text{max}}^\gamma - E_{\text{min}}^\gamma}, \]  

so that the normalisation \( N \) represents the integral flux in the energy range \( (E_{\text{min}}, E_{\text{max}}) \).

The TS value in the table is evaluated as the difference in the log-likelihood obtained in a model with the source and a model without the source. The results are well above 25, which approximately corresponds to a 5\( \sigma \) detection.

The binned analysis was performed dividing the energy range in 8 bins (2 bins per decade). The results of the fits for P7REP and P8 analysis are shown in figure 4.2. In each bin an upper limit\(^2\) at a confidence level of 95\% was evaluated if the TS value of the fit was below 10, which approximately corresponds to a significance of 3\( \sigma \). The lines represent the best-fit power-law distribution obtained from the fit over the full energy range.

The plots do not report directly the photon flux, but the Spectral Energy Distribution (SED) of the source is represented, which is defined as the differential flux multiplied by \( E^2 \). In astrophysics, spectra are usually plotted as SEDs. The physical meaning of the y-axis is an energy flux per logarithmic unit of energy. This representation has the advantage of underlining the features of spectra, which would be otherwise difficult to recognise, just like the cosmic ray spectra shown in chapter 1.

Figures 4.3 and 4.4 show the normalisation of the diffuse models obtained in the binned fit for P7REP and P8 analysis respectively. It can be noticed that the

\( \text{Table 4.3: Results of the fit on the full energy range for P7REP and P8 analysis. See the text for the description of the spectral shape adopted.} \)

<table>
<thead>
<tr>
<th></th>
<th>PASS 7 REPROCESSED</th>
<th>PASS 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N(\text{cm}^{-2}\text{s}^{-1}\text{sr}^{-1}) )</td>
<td>((1.65 \pm 0.47) \cdot 10^{-8})</td>
<td>((1.568 \pm 0.002) \cdot 10^{-8})</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>(-2.05 \pm 0.08)</td>
<td>(-2.04626 \pm 0.00006)</td>
</tr>
<tr>
<td>TS</td>
<td>53</td>
<td>80</td>
</tr>
</tbody>
</table>

\( ^2 \)The upper limit is evaluated when no signal is observed, which means that the null hypothesis of pure background is accepted. In the frequentist approach, the meaning of an upper limit \( U \) at a confidence level \( p \) is the following: if the signal were observed, it would have been above the value of \( U \) with a probability \( p \).
Figure 4.2: Spectral Energy Distribution obtained through the likelihood analysis using P7REP (blue) and P8 (red) Fermi data. The lines represent the best-fit curves obtained from the fit over the full energy range in which a simple power-law was used as a spectral model.

normalisations are distributed around the value of 1, as expected. The Galactic normalisation, which is the dominant one since Tycho is very close to the Galactic plane, is compatible with the value of 1 in all energy bins within a factor of 10%. The other two diffuse models present a greater scattering around this value, which should not create something to worry about since their contribution is much less important than the Galactic diffuse. It must be noted that the Earth Limb is represented only in the first two energy bins, since it does not contribute above 500 MeV, as shown in figure 4.5. In order to validate this statement, the likelihood fit was performed alternatively fitting or fixing the extragalactic diffuse and the Earth limb. The results for the two analysis are shown in figures 4.6 and 4.7. Some discrepancies from the previous results can be noticed at low energy, probably due to a not perfect description of the diffuse emission at low energy. In fact, the diffuse for the P7REP analysis is not optimised for the analysis at low energy. As for the P8 analysis, the reprocessing of data is still ongoing,
hence an improvement of these results is expected. Anyway, these variations must be taken into account when evaluating the systematic errors.

Figure 4.8 shows a comparison of the two analysis, including the systematics evaluated as explained in the previous section. The green points shown in the plot represent the Fermi data points published in [32]. It can be noticed that the results obtained in this work are compatible with the previous data points in the energy range 3 GeV - 100 GeV. The two analysis are also in good agreement for energy above 100 MeV, while a significant drop in the value of the upper limit below 100 MeV is observed for the P8 analysis. This is a direct consequence of the new method of reconstruction and shows the potentiality of this new data reprocessing. The possibility to reconstruct the events with multiple incoming photons time increases significantly (approximately a factor of 3) the effective area of the instrument, above all at low energies, where, being the photon flux higher, the probability of multiple events is much higher.

The importance of the result obtained below 100 MeV, even though it is only an upper limit, will be clear in the next section, since it gives important information about the interpretation of this spectrum.
Figure 4.4: Diffuse normalisations for the P8 analysis. Earth limb is reported only for the first two bins, since it does not contribute to energies above 500 MeV.

Figure 4.5: Energy spectrum of the Earth Limb model.

In order to confirm the goodness of the analysis, a residual map and a TS map were created in both analysis, showing no significant excess in the ROI. Figures 4.9-4.12 show the maps obtained. Residual maps present a uniform distribution, with some very little excess (below 1 σ) corresponding to a few point sources which are far from the center of the ROI and are not fitted in the model. The histograms of residuals show a good agreement with a Gaussian distribution, with some little overall excess (below 10%) which is probably due to an underestimation of the background component.
Figure 4.6: SED obtained through the different fits performed by alternatively fixing or fitting the diffuse models for P7REP analysis.

Figure 4.7: SED obtained through the different fits performed by alternatively fixing or fitting the diffuse models for P8 analysis.
Chapter 4. Evidence of hadronic interaction with Fermi-LAT data

Figure 4.8: Spectral Energy Distribution obtained through the likelihood analysis using P7REP (blue) and P8 (red) Fermi data. Green points are the data points published by the Fermi-LAT collaboration. [32]

The TS maps were obtained after eliminating Tycho from the model, which is the reason why the main source appears at the center of the TS map as a bright spot. For the P7REP analysis the TS of the source is approximately 25, while for the P8 analysis is approximately 60. Some spots with a TS around 10 appear in the P8 analysis TS map, which do not correspond to any known source and can be due to a not perfect description of the diffuse emission.

Figures 4.13 and 4.14 give an enlarged view of this central region. The crosses and the circles indicate the position of Tycho obtained in this analysis, compared to the previous 3FGL position and to the position obtained by the Veritas experiment in the TeV energy range. The green contours are taken from Chandra experiment in the X-ray energy band. As already anticipated at the beginning of this section, there is a significant variation of Tycho coordinates with the new Pass 8 data. The new coordinates are in a very interesting position, since they are in between the Fermi 3FGL coordinates and the ones measured by Veritas and they are perfectly centred with the contours observed in the X-rays.
The localisation of the source was also performed independently in three energy bins, since theoretically the position could change with increasing energy, due to either instrumental (PSF energy dependence) or physical reasons (particular environment of the SNR). In both cases no significant shift was observed with increasing energy.
4.3 SED modeling and discussion

The interpretation of the SED shown in figure 4.8 must be performed through a multi-wavelength analysis of the spectrum. The model to describe the photon spectrum has been developed during the DESY (Deutsches Elektronen-Synchrotron) Summer Student Programme 2013 in which I took part between July 16, 2013 and September 5, 2013 under the supervision of E. Bernardini and R. Gozzini \[33\]. In the following months, it was developed together with M.Caragiulo (PhD student at the University of Bari). The model applied to the Fermi data in \[32\] was presented at the “Cosmic Ray Origin - Beyond the Standard Models” conference
Chapter 4. Evidence of hadronic interaction with Fermi-LAT data

Figure 4.11: TS map obtained from the P7REP analysis. Only photons with energy greater than 1 GeV are taken into account.

Figure 4.12: TS map obtained from the P8 analysis. Only photons with energy greater than 1 GeV are taken into account.
Figure 4.13: Enlarged view of the TS map obtained from the P7REP analysis. Crosses and circles represent the position and relative errors of the source in different energy bins: cyan 1 GeV - 10 GeV; yellow 10 GeV - 100 GeV.

Figure 4.14: Enlarged view of the TS obtained from the P8 analysis. Crosses and circles represent the position and relative errors of the source in different energy bins: blue 100 MeV-1 GeV; magenta: 1 GeV - 10 GeV; red: 10 GeV - 100 GeV.
in S.Vito di Cadore, BL, Italy, from March 16, 2014 to March 22, 2014, and was submitted for proceedings publication. [34]

Data from radio to TeV energy range are taken from different experiments and they are fitted to a model obtained folding the cross sections of the photon production mechanisms described in chapter 2 with the injection spectra of accelerated particles. The free parameters in the injection spectra are obtained from the fit, while the parameters describing the environment of the SNR, such as the magnetic field and the ambient density, are kept fixed thanks to some constraints obtained from radio and X-ray measurements.

Tycho’s distance is not very well constrained. From HI absorption studies [35] it was estimated to be 2.5 to 3 kpc, while measurements of the shocked ejecta velocity [36] report a value around 3 to 5 kpc. Other important parameters useful for the development of a model of the gamma spectrum of this source are the ambient density and the explosion energy $E_{51}$, which are not precisely known but they strictly depend on the source distance. According to [32], the distance was fixed to the value of 3.5 kpc, corresponding to a density of $0.24 \, \text{cm}^{-3}$ and an exploding energy of $2 \times 10^{51} \, \text{erg}$. The low value of the ambient density is the sign of a SNR in an early stage of its evolution, which is not interacting with a molecular cloud. This is one of the main problems in the search for hadronic interaction in young SNRs, as already discussed in section 2.4.1.

As regards the magnetic field, one would expect that the average value of the Galactic magnetic field, which is around $2 \, \mu \text{G}$, should be assumed. However, in many young SNRs, a phenomenon called magnetic field amplification occurs, which enhances its value to few hundreds of $\mu \text{G}$. This amplification is due to the streaming of accelerated particles, which creates a magnetic turbulence well beyond the ordinary value of the Galactic magnetic field. This process has rather solid evidence thanks to the narrow X-ray rims which are widely observed in young SNRs. These filaments are usually interpreted as severe synchrotron losses of high-energy electrons radiating in the strong magnetic field. The width of these filaments allows an estimate for the value of the amplified magnetic field. [37] Tycho is no exception, as it can be noticed from the blue rims in figure 4.15a. For this reason, in this model the magnetic field was fixed to the value of 215 $\mu \text{G}$.

Another evidence of magnetic field amplification in Tycho SNR comes from the observation of a pattern of nearly regularly spaced stripes on the shell surface,
made by the *Chandra X-ray Observatory* [38]. Figure 4.15b shows an image of Tycho SNR in the 4-6 keV range. A detailed analysis of the stripes showed that these filaments can be associated with an emission on the surface of the SNR, i.e. an emission coming directly from the forward shock. The intensity of the X-ray radiation in the stripes was found to be approximately 25 times the average value of the nearby regions and it is not compatible with an enhancement of the emissivity due to a simple projection of the shell geometry. Furthermore, the spacing of these stripes gives information about the maximum energy at which particles are accelerated, which is found to be around $10^{15}$ eV, i.e. just at the *knee* of cosmic-ray spectrum.

### 4.3.1 Injection spectra

As already discussed in section 1.5, Fermi acceleration theory predicts a power-law spectrum for shocked particles, expressed as a function of the kinetic energy of the particle. However, a more complete treatment, generally known as *diffusive shock acceleration* theory, predicts a simple power-law in momentum for the particle density (number of particles per unit volume):

$$n(p) = k \left( \frac{p}{p_0} \right)^{-s}. \quad (4.4)$$

The index $s$ is not kept fixed to the value of 2, as predicted by the Fermi mechanism. In fact, in most cases the value of the index is found to be slightly different from 2. This feature can be explained by the non-linear diffusive shock acceleration theory, in which a modification of the profile of the shock causes a *softening* of the spectrum, which means that the spectral index acquires a value slightly greater than 2.

The intensity of particles is obtained multiplying the density by the particle velocity and dividing by the solid angle:

$$J(p) = \frac{\beta c}{4\pi} n(p). \quad (4.5)$$
Figure 4.15: (A): Tycho SNR in the X-ray and radio energy bands. The blue X-ray rim is a clear evidence of magnetic field amplification. (B): Chandra X-ray 4.0–6.0 keV image of the Tycho SNR, showing various regions of striping in the nonthermal emission. [38]

Since the cross sections of the processes analysed are often expressed as a function on the kinetic energy of the particles, the previous equation can be written as:

\[ J(E_k) = J(p) \frac{dp}{dE_k} = \frac{1}{\beta} J(p(E_k)) \]

\[ J(E_k) \propto (E_k(E_k + 2m))^{-s/2}, \quad (4.6) \]

where \( m \) is the mass of the particle.

Figure 4.16 shows an example of the spectrum of the accelerated particles as a
function of the particle kinetic energy. It can be noticed that the shape is almost
the same as a simple power-law for energy above the mass of the particle $m$, while
a natural break occurs in the spectrum at low energy.

According to these assumptions, both accelerated proton and electron spectra
were modelled with a simple power-law in momentum. However, due to their
mass, electrons suffer from energy losses due to synchrotron emission. This aspect
reflects in a modification of the spectrum at high energy. The correct way to deal
with energy losses involves the resolution of the time evolution equation for the
SNR, which has not been described in this thesis. However, as suggested in [39], a
super-exponential cut-off is a good approximation for the shape obtained from the
full dissertation. Therefore, a super-exponential cut-off at high energy (around a
few TeV) was added in the electron spectrum:

\[
J_e(E_k) = A_e \left( \frac{p(E_k)}{p_0} \right)^{-s_e} \exp \left( - \left( \frac{p(E_k)}{p_{cut,e}} \right)^2 \right)
\]

\[
J_p(E_k) = A_p \left( \frac{p(E_k)}{p_0} \right)^{-s_p}.
\]

The value of the cut-off energy was obtained from the fit.
4.3.2 Results of the fit

The fit was performed independently for the radio and X-ray bands and for the MeV–TeV $\gamma$-rays. The results of the fit are shown in figures 4.17 and 4.18.

The radio to X-ray data are explained by the synchrotron peak, produced by the relativistic electrons deflected in the amplified magnetic field. Once the magnetic field has been fixed, the fit returns the parameters of the electron injection spectrum, i.e. the normalisation, the spectral index and the cut-off energy. From what discussed in section 2.3.1 the electron spectral index is strongly constrained by the slope of the radio measurements, since the following relation between the radio index $\alpha$ and the electron spectral index $s_e$ holds:

$$\alpha = \frac{s_e - 1}{2}. \quad (4.8)$$

The value of the cut-off energy is constrained by the X-ray measurements. Finally, the normalisation depends on the value of the magnetic field, since the higher the magnetic field the brighter the synchrotron emission. The fit returned a best-fit value of the spectral index of $2.34 \pm 0.04$ and a cut-off energy of $(5.1 \pm 0.3)$ TeV.

![Figure 4.17: Spectral Energy Distribution of Tycho SNR from radio to TeV energy range. Red data points are the ones obtained in this work using Pass 8 Fermi data. Other data points are: blue from [40]; purple from Suzaku [41]; green from VERITAS [42]. The dotted, dashed and dotted-dashed lines represent the different contributions to the photon flux, obtained from the fit. The solid line represents the sum of all the contributions.](image)
The other three processes do contribute to the $\gamma$-ray spectrum. However, the constraints obtained from the radio X-ray fit, strictly defines the leptonic contribution, i.e. inverse Compton and bremsstrahlung. The inverse Compton was evaluated considering the scattering of relativistic electrons on both the Cosmic Microwave Background (CMB) photons and the infrared photons coming from dust emission. A blackbody density $n(\omega_0)$ was assumed for both the seed photon populations. CMB photons are characterized by a temperature equal to 2.73 K and an energy density of $0.25 \text{ eVcm}^{-3}$, while, following [37], the temperature and the energy density of the infrared emission were fixed to 100 K and $3.1 \text{ eVcm}^{-3}$ respectively. The calculation showed that in these conditions this contribution does reproduce neither the flux nor the shape of the spectrum. If the magnetic field were one order of magnitude lower, the electron population normalisation obtained from the synchrotron fit would be a factor of 10 greater and the resulting IC contribution could reproduce the $\gamma$-ray flux, but still not the shape. As regards the bremsstrahlung, the low value of the ambient density strongly limits its contribution to the $\gamma$-ray flux, even though models based on different hypothesis on the SNR environment have been proposed.
Being the leptonic contribution negligible, the $\gamma$-ray spectrum can be fitted by the $\pi^0$-decay contribution. The parameters (spectral index and normalisation) of the proton injection spectrum are obtained from the fit. The spectral index is fitted independently from the electron index, even though they are expected to be the same. In fact, the index depends only on the acceleration process, which is the same for both protons and electrons since their mass are negligible with respect to the relativistic energies involved. The best-fit value for the proton population is $2.30 \pm 0.03$ and it is compatible with the electron index within errors.

Once the particle populations are determined, the acceleration efficiency of the shock can be evaluated, assuming that the total kinetic energy expelled during the SN explosion is $2 \cdot 10^{51}$ erg. The total kinetic energy of the accelerated particles is given by:

$$W_{e/p} = \int E_k J_{e/p}(E_k) dE_k.$$  \hfill (4.9)

The $K_{ep}$ value is defined as the ratio of these two values:

$$K_{ep} = \frac{W_e}{W_p}.$$  \hfill (4.10)

Using the best fit parameters, the proton acceleration efficiency (the total kinetic energy of protons normalised to the total explosion energy) is found to be approximately 5%, while the $K_{ep}$ value is approximately $8 \cdot 10^{-3}$.

### 4.3.3 Discussion

As shown in figure 4.18, the hadronic model can explain well the MeV-TeV $\gamma$-ray spectrum of Tycho SNR, reproducing both the flux and the spectral shape. Even though the value found for the spectral index is not exactly 2, as predicted by first-order Fermi mechanism, it can be explained in the frame of the non linear diffusive shock acceleration theory, in which a modified shock produces slightly softer spectra with respect to the Fermi theory. Furthermore, as discussed in section 1.4.4, measurements of the boron-to-carbon ratio, suggest that the cosmic-ray sources must produce a power-law spectrum with an index slightly greater 2, namely between 2.1 and 2.4 and Tycho seems to be exactly in this range.

The value found for the proton acceleration efficiency is also in agreement with the expectations. If SNRs are the main source of cosmic rays in the Galaxy, the energy
Evidence of hadronic interaction with Fermi-LAT data must be compatible with the energy density of cosmic rays measured on the Earth. This energy density can be estimated as follows:

$$\rho_{\text{CR}} = R_{\text{SN}} E_{\text{SN}} \epsilon_p \tau_{\text{esc}},$$  \hspace{1cm} (4.11)

where $R_{\text{SN}}$ is the rate of supernova explosions in our Galaxy, $E_{\text{SN}}$ is the mean energy expelled in each SN explosion, $\epsilon_p$ is the acceleration efficiency, i.e. the fraction of the SN energy transferred to cosmic rays, and $\tau_{\text{esc}}$ is the escape time of cosmic rays, which is approximately 15 Myr. Knowing that $R \approx 2 - 3 S N / (V_{\text{Galaxy}} 100 \text{yr})$ and $E_{\text{SN}} \approx 10^{51} \text{erg}$, an acceleration efficiency of 10% is needed in order to obtain a cosmic ray density of approximately $1 \text{eV cm}^{-3}$.

Also the $K_{\text{ep}}$ value found is in agreement with the expectations. This value is an estimate of the electron-to-proton ratio and can be compared to the one observed at the Earth, which is approximately $10^{-2}$.

All these results suggest that the model described so far can correctly reproduce the data satisfying all the request to support the SNR paradigm for CRs, although very few assumptions have been made. Hence, Tycho SNR represents a very special case to test the acceleration theory and have direct evidence of cosmic-ray acceleration in SNRs.

More complete models have been developed to support the hadronic scenario, which are in good agreement with the results found with this model (see for example [37]). On the other hand, other models supporting the leptonic scenario have been developed, making some more complicated assumptions on the SNR environment. For example, in [43] a two-zone model was developed in which the bremsstrahlung emission was assumed to come from two different regions with two values of ambient density. Figure 4.19 shows the result of this model compared to the previously published Fermi-LAT data. It can be noticed that this model is in agreement with the old Fermi data [32], but does not correctly represent the spectral shape found from the Pass 8 analysis developed in this work. In fact, the low energy upper limits found in this analysis are in good agreement with the shape arising from the pion-decay emission, strongly favouring the hadronic origin of $\gamma$-rays.

Anyway, I must remark that the Pass 8 results found in this analysis are still preliminary, since the data reprocessing is still ongoing within the collaboration.
Chapter 4. Evidence of hadronic interaction with Fermi-LAT data

Figure 4.19: $\gamma$-ray flux of Tycho SNR described with a leptonic model. The heavy solid line shows the total flux of leptonic origin. For more details see [43].

In conclusion, Tycho is the ideal case to look for evidence of the acceleration of cosmic rays in SNRs, and provides a good test for the Fermi acceleration theory. However, this is only the starting point in order to work out the question of the origin of cosmic rays. Many problems are still present between the experimental data and the theory. One of the most difficult question to answer is the maximum energy of acceleration. For example in the Tycho’s case the maximum energy for proton was fixed at a value of few hundreds TeV, since higher values of proton energy contribute to create $\gamma$-rays with an energy higher than the maximum energy detected by Veritas. Hence, there is still no evidence that SNRs can accelerate cosmic rays up to $10^{15}$ eV, where the break in the CR spectrum occurs. In this sense, future experiments like the Cherenkov Telescope Array (CTA), which is currently under development, will give information with an increased sensitivity also at very high energies.
Conclusions

The main goal of the work in this thesis was the search for evidence of hadronic interaction in Supernova Remnants, which is a clear sign of the acceleration of cosmic rays in SNRs. Being CRs deflected by the galactic magnetic field, the only way to trace their presence at the source is through the detection of the products of their interaction with the environment of the source. In particular, accelerated protons interact with target protons and nuclei around the source and produce γ-rays via neutral pion decay. Therefore, the study of γ-ray spectra of the sources may give information about this interaction.

The study in this thesis was aimed to the determination of the γ-ray spectrum of Tycho SNR using Fermi-LAT data. The analysis was conducted using the new reprocessed Fermi-LAT data (Pass 8 data), which are characterised by a new method of reconstruction that produces an enhancement of the effective area, especially at low energy. The analysis performed also contributed to the process of validation of these data within the Fermi-LAT collaboration.

The spectrum obtained was then interpreted through the development of a model that, given a spectral shape for the accelerated particles, predicts the photon spectrum from radio to TeV energy range, taking into account the most important non-thermal photon emission processes.

The results obtained showed that the spectral shape of the γ-ray spectrum was compatible with the emission coming from the π⁰ decay, supporting the idea of hadronic interaction in the SNR. It was also shown that Pass 8 data do have a key role to reach this conclusion, since they give important information about the spectral shape at energies below 100 MeV.

The model developed was in good agreement with the requirements of the acceleration theory in order to support the SNR paradigm for CRs. Furthermore, the
multi-wavelength study of Tycho showed that this SNR is a very good example to test the acceleration theory in all its details.

However, it must be underlined that the results obtained with Pass 8 data are still preliminary, and so they cannot be considered conclusive. In the forthcoming future, the Pass 8 reprocessing will be refined and more conclusive results will be available. If confirmed, this analysis will for sure provide strong hints of the acceleration of cosmic rays in SNRs and be the starting point for further investigations on the CR acceleration in SNRs.
Appendix A

The spectral index of the synchrotron emission

As already discussed in section 2.3 the typical shape of the synchrotron emission covers the radio to X-ray energy band and is peaked\(^1\) around a value which depends on the maximum energy of the relativistic electrons.

One of the most important features of this “peaked” distribution is that a power-law spectral shape for the electron population results in a power-law photon spectrum up to the maximum value of the distribution and the two spectral indices are strictly related:

\[
\alpha = \frac{s - 1}{2},
\]

where \(s\) is the spectral index of the electron population and \(\alpha\) is the one of the synchrotron emission. This result is very important because few measurements in the radio band give strong constraints on \(s\).

In principle this result can be derived assuming a power-law distribution for \(dN/d\gamma\) and solving the integral in equation (2.10). However, it can be derived in a much simpler way.

As can be seen in figure [A.1] the synchrotron spectrum emitted by the single electron is peaked. Then, the emission at frequency \(\nu\) can be supposed to depend only on electrons with Lorentz-factor \(\gamma\), so that their single-electron spectrum

\(^1\)The typical shape of the photon flux is always a decreasing curve and no peak is visible. The “peak” appears when the flux is represented in the form of Spectral Energy Distribution (SED), i.e. it is multiplied by the factor \(E^2\).
peaks at $\nu$. According to equation (2.7), the relation between $\nu$ and $\gamma$ is: $\nu = \gamma^2 \nu_L$.

To simplify, assume that the pitch angle is $90^\circ$. From equations (2.2) and (2.4), the power emitted by a single electron of Lorentz factor $\gamma$ is:

$$P_e = \frac{2e^2}{3c^3} a_\bot^2 = \frac{2e^2}{3c^3} \gamma^4 \left( \frac{evB \sin \theta}{\gamma mc} \right)^2 = \frac{2e^4}{3m^2c^3} \gamma^2 \beta^2 B^2. \quad (A.2)$$

$$Be \frac{dN}{d\gamma} \propto \gamma^{-s},$$

the emissivity from the electrons with Lorentz factor $\gamma$ is:

$$\epsilon_s(\nu)d\nu \propto P_e \frac{dN}{d\gamma}d\gamma \Rightarrow \epsilon_s \propto P_e \frac{dN}{d\gamma} \frac{d\gamma}{d\nu}. \quad (A.3)$$

From the relation $\nu = \gamma^2 \nu_L$, one obtains:

$$\nu = \gamma^2 \nu_L \Rightarrow \gamma = \left( \frac{\nu}{\nu_L} \right)^{1/2} \Rightarrow \frac{d\gamma}{d\nu} = \frac{1}{2} \frac{\nu^{-1/2}}{\nu_L^{1/2}}. \quad (A.4)$$

Hence, recalling that $\nu_L \propto B$ and assuming $\beta \approx 1$:

$$\epsilon_s \propto \gamma^2 B^2 \gamma^{-s} \nu^{-1/2} B^{-1/2}. \quad (A.5)$$
Finally, from equation (A.4):

\[ \epsilon_s \propto \left( \frac{\nu}{\nu_L} \right)^{(2-s)/2} B^{3/2} \nu^{-1/2} B^{s/2} \nu^{-1/2} = B^{(s+1)/2} \nu^{-(s-1)/2}. \] (A.6)

If \( \epsilon_s \propto \nu^{-\alpha} \), then:

\[ \alpha = \frac{s-1}{2}, \quad \text{q.e.d.} \] (A.7)
Bibliography


