Study on $J/\psi$ from beauty hadrons at $\sqrt{s_{NN}} = 5.02$ TeV in p-Pb collisions at ALICE-LHC
# Contents

Introduction iii

1 Collider Physics at LHC Energies 1
   1.1 Characterization of hadronic collisions .......... 3
      1.1.1 Centre of Mass Energy ...................... 3
      1.1.2 Rapidity .................................. 5
      1.1.3 Centrality .................................. 6
   1.2 New feature at LHC : accessible x range ........... 9
   1.3 Deconfined phase in High-Energy Heavy-Ion Collisions .... 12
      1.3.1 The QGP Phase Transition ................... 13
      1.3.2 Stages of Heavy-Ion collisions ............... 15
      1.3.3 Evidences of a new state of matter ............ 17
   1.4 Study of Heavy-Ion Collisions ..................... 21

2 J/ψ Meson in Heavy-Ion Collisions 25
   2.1 The Charmonium Family ........................... 26
   2.2 Charmonium Production in Hadronic Collisions ......... 29
      2.2.1 Heavy quark pair production .................. 30
      2.2.2 Charmonium formation models ................. 31
   2.3 Charmonium Production and Absorption in Nuclear Medium ... 35
      2.3.1 Charmonia in proton-nucleus collisions ......... 35
      2.3.2 Charmonia in Nucleus-Nucleus collisions ........ 43
   2.4 J/ψ and Measurement of B Hadron Production .......... 48

3 The ALICE Experiment at LHC 51
   3.1 Detectors Layout .................................. 51
Introduction

Ultrarelativistic heavy-ion collisions have shown that nuclear matter dynamics changes as a function of the energy density. Our Universe during the first few microseconds after the *Big Bang* was in an an extreme energy density regime and at very high temperatures, therefore exploring extreme conditions as the ones reached in nuclei collisions at LHC energies can provide us a better understanding of the first moments of the birth of the Universe. The theoretical tool which provides predictions on the dynamics of QCD matter under this new regime is the lattice QCD. The theory has shown that above a critical value of energy density and temperature, the colliding hadronic system undergoes a phase transition, creating an ensemble of deconfined hadronic matter, namely the Quark Gluon Plasma or QGP. Under such extremely high-energy density conditions, *quarks* and *gluons*, the most fundamental known constituents of all surrounding visible matter, are expected to be no longer confined into small hadrons but rather to act as quasi-free particles into a larger volume full of colored partons, such as charge particles in electromagnetic plasmas. After the formation of such system, pressure gradients make the colliding system expanding and cooling until the conditions for the formation of hadronic matter are reached. Acquiring direct experience of the formed QGP phase is an experimental challenge, since the detected hadrons undergo several processes before decoupling from the hot medium (e.g.: final hadronic rescattering). It is fundamental, then, to correctly evaluate and discriminate the signatures of the hot plasma from other still not fully understood “colder” effects which take place in the complex evolution of heavy nuclei collisions. Lot of focus has thus been placed in the search for the best suited experimental tools to probe such a short-living and hidden phase. Heavy *quarkonia* states, such as $J/\psi$ were the first signals of the new QCD regimes. The $J/\psi$
production was suppressed in heavy ion collisions with respect to pp collisions and this was considered a sign of the formation of a plasma of hot partons. Ever since 1986, much experimental effort has been dedicated in order to reproduce this kind of “little bang” in the laboratory, and the first strong evidences that a state of deconfinement could indeed have been observed came only several years later from the SPS and RHIC programmes. The startup of LHC experiments at CERN in 2009 has brought heavy ion physics research into a higher and unreached energy domain, allowing both more data to be collected as well as more constraints to be put in the broad environment of theoretical predictions. ALICE experiment in particular, followed by ATLAS and CMS, was optimized to perform these kind of analysis and has so far collected measurements on lead-lead and, more recently, on proton-lead collision events.

The thesis work will develop an analysis performed on a data sample of $J/\psi$ candidates collected by the ALICE collaboration in proton-lead collisions at centre of mass energy $\sqrt{s_{NN}} = 5.02$ TeV per nucleon collision. Aim of the analysis has been the extraction of the fraction of the so-called “non-prompt” component of the yield, made up of all those $J/\psi$ produced after the decay of a heavier beauty hadron and thus relatively displaced from the primary interaction vertex. The analysis is a benchmark for heavy flavour and quarkonium production in heavy ion collisions as well as the one performed in proton-proton since they can provide useful insights in cold nuclear matter effects.

The first chapter will summarize the main features of ultra-relativistic ion physics at LHC. An highlight will then be given, throughout the chapter, to the experimental characterization of nucleon-nucleus collisions as well as to their fundamental role as reference measurements for the extrapolation of cold nuclear matter effects in heavy-ion collisions. The second chapter will be dedicated to quarkonia states and in particular to the $J/\psi$ charmonium state. After a qualitative overview of the most widely used production models in heavy-ion physics, a more detailed description of their role as information-carrier of the QGP phase will be given. A review of the most important as well as of some more recent LHC experimental measurements will also be presented, with a final highlight to their use as indirect
measure of beauty quark pairs.

In the third chapter, a short description of the ALICE experimental apparatus will be provided and the main features of the ALICE will be described. A highlight will be given to central barrel detectors, such as the *Inner Tracking System* (ITS) detectors and the *Time Projection Chamber* (TPC). Some insights will also be provided on secondary vertex reconstruction and on particle identification methods for electrons, relevant for the understanding of the data sample analysed in this thesis work.

The fourth and fifth chapters will finally be devoted to the detailed exposition of the performed analysis. At first, information will be given regarding the acquisition of Minimum Bias data sample. Then, the full explanation of the non-prompt $J/\psi$ fraction extraction technique and of the obtained results will be provided. Fifth chapter will discuss the obtained results and explicit the methods employed for the estimation of the statistical uncertainties.
Chapter 1

Collider Physics at LHC Energies

All our understandings of the innermost constituents of sub-atomic matter, such as the structure of nucleons inside nuclei and the proprieties of all the other unstable particles, couldn’t be reached without the technological achievements of particle colliders. The higher the speed of the probing particle, the more information its interactions will provide on smaller constituents of the target, up to the limit in which the whole process may be described in terms of elementary quarks and gluons.

An essential characteristic of a high-energy collision between two nucleons is that a large fraction of the projectiles kinetic energy will be dissipated during the collision in a hard collisions. During an elementary collision, then, the dominant part of the total nucleon-nucleon cross section $\sigma$, relies in its inelastic in nature [1].

Roughly speaking, this means that, if a collision occurs, the projectile and target will be most likely to change their nature rather than to simply “bounce off” of each other. Most of the released energy will form hadrons (such as pions or more rare mesons), leptons and photons, which will eventually be detected by the detectors. What I mean to stress in this very simplified picture is that

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The cross section $\sigma_i$ of an elementary process $i$ is dimensionally equivalent to an area of the typical size of fractions of barns. It can be defined as the ratio between the observed process rate $\frac{dN_i}{dt}$ and the so-called luminosity $L$ of the system: $\sigma_i = \frac{dN_i}{L}$. 

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the more energy is provided to the colliding particles, the more will be the ways, or “channels”, available for its release. These are substantially the main reasons which explain why, in the past decades, billions of dollars were spent to build increasingly large and powerful particle accelerators all over the world, either with higher maximum energies or with higher luminosities, under the common aim of revealing more about the fundamental pieces of our Universe.

The first ever built electron-positron collider was the Italian AdA (Anello di Accumulazione) at Frascati, to which many others followed with an exponential increase in their energy reach over the years [2]. Before the startup of LHC (Large Hadron Collider) by CERN (European Organization on Nuclear Research) in 2008, the highest available energies were those of the Tevatron at Fermilab (Fermi National Accelerator Laboratory), with protons and anti-protons beams accelerated to energies of up to 1 TeV, and of RHIC (Relativistic Heavy Ion Collider) at Brookhaven, with beams of heavy gold nuclei accelerated up to the energies of 200 GeV per nucleon. LHC pushed particle physics into a whole new energy domain with an astonishing increase up to the predicted maximum reach of 14 TeV for protons in the centre of

Figure 1.1 – Colliders centre of mass energy over the years, from [2].
mass. This achievement will not only, as already mentioned, be valuable for the innermost probing of nucleons structures, but also for the measurement of physical effects at unprecedented scales, which will provide more constraints for Quantum Chromo Dynamics (QCD) models. In particular, the performed and programmed heavy-ion collisions of lead nuclei up to the predicted energies of 5.5 TeV per nucleon pair in the centre of mass, will widely extend the frontiers of research in the field of the Quark Gluon Plasma physics.

Aim of this chapter will be that of giving a proper contextualization of the performed analysis in this constantly developing environment which is that of high-energy physics at colliders, outlining in particular what are the key aspects of nucleus-nucleus and proton-nucleus collisions in view of the complex physics of QGP related phenomena.

1.1 Characterization of hadronic collisions

A new set of dedicated observables are used ultrarelativistic heavy ion collisions. They allow for a comparisons of measurements at different energies and at different colliding systems. The ones described in the next sections are: center of mass energy per nucleon pair ($\sqrt{s_{NN}}$), the sistem rapidity and the collisions centrality

1.1.1 Centre of Mass Energy

Estimating the energy of the colliding system is a first fundamental step to characterize the dynamics of particle interactions. The total squared energy $s$ evaluated in the centre of mass system of two colliding particles is a Lorentz invariant observable which quantifies the maximum energy at disposal for the system for its processes, such as nucleon excitations or particle production. I’ll start by introducing how to compute $s$ in the simple case of two colliding nucleons.

Let’s consider two massive particles $m_1$ and $m_2$ with relativistic four-momentum vectors $p_1 = (E_1, \mathbf{p}_1)$ and $p_2 = (E_2, \mathbf{p}_2)$ in the laboratory frame. For each particle, the energy in the laboratory frame is just the first component of their four-momentum vector:
The squared total energy in the centre of mass system may then be easily proven to be equal to the squared norm of the total four-momentum \( p_1 + p_2 \), so that

\[
\sqrt{s} = \sqrt{(p_1 + p_2)^2} = \sqrt{(E_1 + E_2)^2 - (p_1 + p_2)^2} = E_{cm} \quad (1.1)
\]

In a fixed target experiment, one of the two nucleons is at rest, e.g. \( p_2 = (m_2,0) \), and the centre of mass energy in the ultra-relativistic limit is \( \sqrt{s} = \sqrt{(m_1^2 + m_2^2 + 2m_2E_1)} \approx \sqrt{2m_2E_1} \). A collider ring such as LHC instead has the particles travelling in opposite directions. In the case of two identical particles, like two protons, the total energy equals the centre of mass squared energy \( (E_1 + E_2)^2 = (2E)^2 = s = E_{cm}^2 \). This means that all the energy furnished to the beams is available in the centre of mass frame, and so that less energy has to be provided to the beams in order to reach the same value of \( \sqrt{s} \) with respect to the analogous fixed target case. This fact well explains the rapid development of these kind of experimental apparatuses in high-energy physics.

The computation made for the two nucleons can be extended to the general case of two different nuclei with different mass and charge numbers. Instead of the total centre of mass energy, a more meaningful observable is the centre of mass energy per nucleon pair \( \sqrt{s_{NN}} \), that is also easily comparable to the proton-proton case.

Let the two protons be circulating in opposite direction inside a collider, their four-momentum is \( p \) and \( E = \sqrt{s}/2 \). The electromagnetic field of the apparatus provide acceleration only to the charged nucleons, therefore if two nuclei are accelerated instead of protons, the resulting four-momenta of the accelerated nucleons inside the nuclei will than be scaled by a fraction \( \frac{Z}{4} \) with respect to the previous case.

For two nuclei with charge and mass numbers of \( (Z_1, A_1) \) and \( (Z_2, A_2) \)
respectively, the four momentum vectors of the nucleons will then be

\[ p_1 = p(Z_1, A_1) = \frac{Z_1}{A_1} p^p \]

\[ p_2 = p(Z_2, A_2) = \frac{Z_2}{A_2} p^p \]

and within the limit of ultra-relativistic collisions the centre of mass energy per nucleon pair can be written as

\[ \sqrt{s_{NN}} = \sqrt{(p_1 + p_2)^2} \simeq \sqrt{4|p_1||p_2|} = \sqrt{\frac{Z_1 Z_2}{A_1 A_2}} \sqrt{s^p} \quad (1.2) \]

As conclusive example, we will compute the energy per nucleon pair for the ALICE proton-lead data acquisition run taken in analysis for this thesis work. Since for the proton-proton runs \( \sqrt{s^p} \) amounted to 8 TeV, considering for lead \( A = 208 \) and \( Z = 82 \) whereas for proton \( A = Z = 1 \), we can calculate \( \sqrt{s_{NN}} \simeq \sqrt{\frac{82}{208}} 8 \text{ TeV} \approx 5.02 \text{ TeV} \).

### 1.1.2 Rapidity

In characterizing the kinematic processes of heavy-ion collisions as well as many other high-energy phenomena, it is very convenient to utilize variables which possess similar proprieties under a change of the frame of reference. A fundamental variable used to characterize the momentum distributions of the reaction products in terms of their disposition with respect to the colliding beams direction is the *rapidity* \( y \).

The *rapidity* \( y \) of a particle in the laboratory frame, is defined in terms of its four-momentum components by

\[ y = \frac{1}{2} \ln \left( \frac{E + p_L}{E - p_L} \right) \quad (1.3) \]

where \( E \) is the energy of the particle and \( p_L \) indicates the longitudinal component of the particle momentum with respect to the beam direction. Rapidity actually does depend on the chosen frame of reference. In considering different frames of reference accelerated in different directions, one can be easily imagine that the spread of the momentum distribution as described by the rapidity variable will change, nonetheless this frame of reference dependence
is very simple in the common case of Lorentz boosts along the beam direction. It can be easily shown that if one considers a frame of reference boosted with velocity $\beta$ in the beam direction, the rapidity $y'$ in the new frame can be related to the rapidity $y$ of the old frame by

$$y' = y - y_\beta$$ (1.4)

where

$$y_\beta = \frac{1}{2} \ln \left( \frac{1 + \beta}{1 - \beta} \right)$$ (1.5)

represents the rapidity of the moving frame.

This addition propriety of rapidity reveals particularly useful when relating with asymmetrical collisions, as in the case of different masses of the projectiles or, more generally, of different momentum beams. Under this circumstances, in which the centre of mass is not at rest in the laboratory frame, one can easily compare the results of different experiments by referring to the rapidity distributions in the centre of mass frame. Physically this translates only with a shift, expressed by (1.5), of their distribution given by the rapidity of the moving centre of mass.

As a relevant example for this work purposes, we can consider the actual case of a proton-lead collision with $\sqrt{s_{NN}} = 5.02$ TeV. Here, protons travel with a momentum $p_p = 4$ TeV/c whereas lead nucleons have a momentum of $p_{Pb} = \frac{Z}{4} 4$ TeV/c $\approx 1.58$ TeV/c. The nucleon-nucleon centre of mass of the system moves then with a velocity $\beta_{NN} \approx 0.434$ in the proton direction which means one has to account for a rapidity shift of $y_{\beta_{NN}} \approx 0.465$ in the proton beam direction.

### 1.1.3 Centrality

The most evident aspect one has to account when comparing nucleon-nucleon to nucleus-nucleus and nucleon-nucleus collisions, is that nuclei are composite many-nucleon systems, which implies some way that nucleus-nucleus collisions involve the dynamic of multiple colliding nucleons. The characteristics of these interactions are much more complex than for the relatively simple case of two colliding protons, and one therefore must take into account that the geometry of the process determines in a large way the observed results.
A *peripheral* collision will surely imply that less nucleons are participating with respect to the case of *central* collision. A quantification of many relevant aspects can be done then by evaluating the which is the *impact parameter* of the colliding system, defined as the length of the vector conjugating the two colliding nuclei. A simplified picture of the process can be sketched from figure 1.2. The underlying concept is that the impact parameter $b$ actually deter-

![Figure 1.2](image-url)  

**Figure 1.2** – Qualitative representation of a heavy-ion collision in terms of participant and spectator nucleons. The two Lorentz-contracted nuclei collide with impact vector $b$, whose length represents the impact parameter. Image from [3].

mines the number of *participants* and *spectator* nucleons in the collision, that is to say which nucleons will hit the nucleons of the other nucleus. Experimentally one could thus guess the grade of *centrality* of a nuclear collision by evaluating the fraction of energy carried by the spectators and deposited in some *Zero Degree Calorimeters* (ZDC) or by looking at the total multiplicity of the detected particles, which is expected to increase with the number of participants. Both of these quantities may in principle be used to reconstruct the process impact parameter and thus extract the very important class of *central collision events*, which are namely those with $b \simeq 0$. This is, in particular, of fundamental use for a *rescaling* of the observed data to the proton-proton collision case, (in which obviously only two participant nucleons are present) and also for a quantitative comparison of different heavy-ion collisions [3].

A thorough study on the statistical relations related to the geometry of nuclear collisions in terms of number of participant nucleons and of binary nucleon-
Chapter 1: Collider Physics at LHC Energies

nucleon collisions was made by R.J. Glauber [4]. By employing realistic nuclear density distributions and nucleon-nucleon cross sections, Glauber’s model allows to estimate the average number of participant nucleons and of binary collisions, alongside with their statistical uncertainties, as function of the impact parameter. The latter, in particular, is necessary for quantitative comparisons with proton-proton collisions. From an experimental point of view, assuming a monotone dependence of some measurable quantity \( n \) (such as the charged multiplicity, energy in ZDC’s, number of participants or of binary collisions etc.) to the grade of centrality (namely, to the impact parameter) of the collision \( (n = n(b)) \), one usually divides the data sample in centrality classes \( c(N) \), defined as the percentile of events with highest \( n \) for which \( n > N \).

The evaluation of centrality for proton-Nucleus collisions case is not as straightforward as it would look from this brief introduction. For instance, the number of participant nucleons \( N_{\text{part}} \) is more poorly correlated to both the impact parameter and the multiplicity of charged particle with respect to the nucleus-

**Figure 1.3** – Measured distribution of reconstructed charged tracks multiplicity in the ALICE TPC detector, divided in different centrality classes. Lower percentile classes correspond to lower values of impact parameter and thus to more central collisions. Notice the distribution is fitted with the predicted distribution from Glauber’s model, for which it accounts a negative binomial distribution (NBD) proportionality both to the number of participants \( N_{\text{part}} \) and of binary nucleon collisions \( N_{\text{coll}} \).
Chapter 1: Collider Physics at LHC Energies

nucleus case. However, provided some thoughtful choices of the categorizing variable, it is still possible to divide data samples in reasonable centrality classes.

When measurements are taken averaging over all the centrality classes, and thus over different values of impact parameters, one speaks of minimum-bias data. That is the case, for example, of the extracted data sample analysed in this work.

1.2 New feature at LHC: accessible x range

To get a quantitative estimate of the high-energy effects implied in LHC collisions, let’s consider the case of the production of a heavy quark pair $Q\bar{Q}$ from the inelastic collision of two nucleons.

In perturbative QCD, heavy particle production from inelastic nucleon collisions can be pretty well described in terms of a hard interaction between two partons from the respective nucleons. Bjorken $x$ variable is usually defined as the fraction of the nucleon’s momentum carried by the parton which enters in the hard scattering process. It is a measure of the degree of “inelasticity” of the collision and its accessible range is strictly correlated to the four-momentum $Q^2$ (sometimes called virtuality) transferred during the collision and to the available energy in the nucleon-nucleon centre of mass frame. The distribution of $x$ for a given parton (like a valence quark, a sea quark or a gluon) is called Parton Distribution Function (PDF) and describes the partonic composition of the nucleon at at the “resolution scale” defined by $Q^2$.

If we take the case of the production of a heavy quark pair $Q\bar{Q}$ (such as a $c\bar{c}$ or $b\bar{b}$ pair), at leading order in QCD perturbative approach it can be viewed as coming from a gluon-gluon fusion process $gg \rightarrow Q\bar{Q}$. For the general case of the two gluons coming from two ions $(A_1,Z_1)$ and $(A_2,Z_2)$ with centre of mass energy per nucleon pair $\sqrt{s_{NN}}$, their four-momenta will be equal to

$$(x_1,0,0,x_1)\frac{Z_1}{A_1}\sqrt{s^p}$$
Chapter 1: Collider Physics at LHC Energies

\[(x_2, 0, 0, x_2) \frac{Z_2}{A_2} \sqrt{s^p},\]

with \(\sqrt{s^p}\) being their corresponding p-p collision c.m.s. energy.

Some calculations easily allow to evaluate the square of the invariant mass \(M_{Q\bar{Q}}^2\) of the \(Q\bar{Q}\) pair:

\[M_{Q\bar{Q}}^2 = x_1 x_2 s_{NN} = x_1 x_2 \frac{Z_1 Z_2}{A_1 A_2} s^p, \tag{1.6}\]

and its longitudinal rapidity \(y_{Q\bar{Q}}\) in the laboratory frame:

\[y_{Q\bar{Q}} = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right) = \frac{1}{2} \ln \left( \frac{x_1 Z_1 A_2}{x_2 Z_2 A_1} \right). \tag{1.7}\]

By combining these two equations one can then derive the dependence of \(x_1\) and \(x_2\) on \(A, Z, M_{Q\bar{Q}}\) and \(y_{Q\bar{Q}}\):

\[x_1 = \frac{A_1 M_{Q\bar{Q}}}{Z_1} \exp(+y_{Q\bar{Q}}), \tag{1.8}\]

\[x_2 = \frac{A_2 M_{Q\bar{Q}}}{Z_2} \exp(-y_{Q\bar{Q}}), \tag{1.9}\]

so that it is possible to compute the accessible \(x\) range for a reaction with a certain \(M_{Q\bar{Q}}\) threshold and within given experimental rapidity acceptance cuts.

For the production of a bottom \(b\bar{b}\) quark pair, in a p-p collision at the LHC maximum energy of 14 TeV, the probed \(x\) at central rapidities, where \(x_1 \simeq x_2\), is of \(\simeq 6.4 \cdot 10^{-4}\); whereas for a charm \(c\bar{c}\) quark pair, due to its lower mass, \(x\) values go down to \(\simeq 1.7 \cdot 10^{-4}\). Forward rapidities, moreover, allow further smaller values of \(x\) to be probed: the forward \(y \simeq 4\) rapidity region, gives access to \(x\) regimes about 2 orders of magnitude lower, down to \(x \sim 10^{-6}\).

Figures (1.4) and (1.5) clarify the reported formulations by picturing the accessible \(x\) ranges in a \((x_1, x_2)\) plane in the case of the ALICE experimental apparatus at LHC, for p-p, p-Pb, and Pb-Pb collisions.

In the log-log scale, either the constant rapidity exponentials points \((x_1 = x_2 \exp(+2y_{Q\bar{Q}}))\), or the constant invariant mass hyperbolas points \((x_1 =\)
\[ M_{QQ}^2/(x_2 s_{NN}) \], lie on straight lines in the plane. Those corresponding to the production of \( c\bar{c} \) and \( b\bar{b} \) pairs at the threshold are also showed; Shadowed regions represent the acceptance of the ALICE central barrel, covering the pseudo-rapidity range \(|\eta| < 0.9\), and of the muon arm, with \(2.5 < \eta < 4\).

In the case of asymmetric collisions (1.5), like p-Pb and Pb-p, a rapidity shift \( \Delta y = 0.42 \) has to be taken into account, as already explained in section (1.2.2). These figures do not take into account that rapidity and \( p_T \) cuts for the actual detected particles coming from the heavy quark pairs will produce some increase in the minimum experimentally probed invariant mass and consequently to the effective \( x \) interval, but are, however, not a too drastic approximation and may give a significant idea of what are the unprecedented low \( x \) values reached in hadronic collisions at LHC energies. If compared to previous experiments, like SPS or RHIC, the \( x \) regime relevant to charm production at the LHC (\( \sim 10^{-4} \)) is about 2 orders of magnitude lower than at RHIC and 3 orders of magnitude lower than at SPS.

The conceptual relevance of this new probed regime is noticeable. Although very small values of Bjorken \( x \), as low as \( \sim 10^{-5} \), have been already reached in the deeply inelastic e-p collisions at HERA, in those experimental constraints they were associated with rather small values of the transferred momentum \( Q^2 \), (about \( Q^2 < 1 \text{ GeV}^2/c^2 \) for \( x < 10^{-4} \)) which are only marginally under

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure14.png}
\caption{ALICE acceptances in the Bjorken \((x_1,x_2)\) plane for Pb-Pb (left) and p-p (right) collisions at \( \sqrt{s_{NN}} = 5.5 \) and 14 TeV respectively. Charm and Beauty quark pairs invariant mass production thresholds are reported. Figure from [10].}
\end{figure}
control in perturbation theory. At LHC energies instead, most of the produced particles is already controlled by partons with $x \sim 10^{-3}$ and, under the right kinematical conditions, it will be possible to reach values as low as $x \sim 10^{-6}$ with truly “hard” momentum transfers as high as $Q^2 = 10 \text{ GeV}^2/c^2$ [11] (recall that for our leading order heavy flavour production process $Q^2 = M^2_{\bar{Q}Q}$).

Because of these reasons, LHC capabilities open doors for a more conclusive check of our theoretical understanding of the nature of nucleons and nuclei at low $x$, and, most important for heavy-ion physics, allows a structured investigation of many physical effects, like gluon saturation and nuclear shadowing which I will discuss in the following sections, which are expected to strongly characterize the parton distribution functions of nucleons inside nuclei at these energies.

1.3 Deconfined phase in High-Energy Heavy-Ion Collisions

The main actual quest of heavy-ion physics surely lies in the extensive research on theoretical findings of a state of deconfinement for strongly interacting matter which has been plausibly observed experimentally in the most
recent high-energy heavy-ion collisions. This section will provide some short insights on the phenomenological aspects of this puzzling feature of hot and dense matter.

1.3.1 The QGP Phase Transition

On the basis of quantum chromo-dynamics, we can consider all strongly interacting elementary particles as bound states of point-like quarks. All quarks show themselves as “confined” into hadrons by a binding potential which increases linearly with their relative separation. Hence, it is not possible to split an isolated hadron into its quark constituents since an infinite amount of energy would be needed to isolate each quark.

Already before the discovery of quarks and the establishing of QCD as the fundamental theory of strong interactions, observations based on a thermodynamical description of hadronic resonance yield and mass distributions hinted some kind of critical behaviour of hadronic matter at extremely high temperatures of the order of 160 - 180 MeV [13]. The subsequent formulation of QCD led Cabibbo and Parisi in 1975 [14] to identify this temperature with that of a transition from ordinary hadronic matter to a new phase in which all quarks and gluons degrees of freedom became available. The first phase diagram of strongly interacting matter was sketched and, three years later, the term “quark-gluon plasma” was introduced by Shuryak [15] to identify the new phase. With the development of computational science and of lattice QCD simulations in the early 80’s, calculations taking partially into account non-perturbative effects were made possible and quantitative predictions about the proprieties of the newly found state started to take place in the increasingly fervent theoretical scenario. Most practical limitations to lattice simulations still arise from the fact that they do not usually include a finite baryo-chemical potential (a measure of net baryonic density), nor take into account the masses of the quarks. Nonetheless, all results clearly showed how at really high densities and/or temperatures, at an approximately constant value of energy density of about $\simeq 1 \text{ GeV/fm}^3$, a transition to a plausibly deconfined phase occurred.

Figure (1.6) shows, as meaningful example, one of the first lattice computa-
Figure 1.6 – Lattice calculations for the energy density divided by the fourth power of temperature, for different values of quark simulated masses. Arrows indicate the maximum expected initial temperatures reachable by different accelerator facilities. Data from [16].

tions results by Karsch et al. [16] for the value of the energy density over the fourth power of temperature $\epsilon/T^4$ as function of temperature. Results were obtained by assuming a zero baryo-chemical potential and by considering the case of two-flavoured and three-flavoured QCD with massless quarks, plus a more realistic one with two massless light quarks and one finite-mass heavier quark. Even if not of the “first-order” type, a rapid continuous transition at about $T_c \simeq 175$ MeV rises up the value of $\epsilon/T^4$ of about 8 units over a small temperature range. A very similar picture is moreover observed for other thermodynamical observables like pressure or entropy. Since classically, in the limit of an ideal Stefan-Boltzmann gas, the energy density $\epsilon$ is related to the temperature by the number of basic degrees of freedom $n_{dof}$ by

$$\epsilon = n_{dof} \frac{\pi^2}{30} T^4,$$ \hfill (1.10)

the rapid increase in energy density at the transition point can be interpreted as large increase in the effective number of degrees of freedom, from that of a typical pion gas, to that of a deconfined quark-gluon plasma in which colour degrees of freedom are newly available².

²The value of $n_{dof}$ is computed by appropriately summing over the number of flavours $\times$ spin $\times$ quark/anti-quark $\times$ colour for quarks and over the number of polarizations $\times$ colours
1.3.2 Stages of Heavy-Ion collisions

The only way to achieve extreme energy densities is still now to collide two ultrarelativistic heavy nuclei and then analyse the resulting traces of the early short-lived medium. This brought to the start of the heavy-ion experimental programme in 1986, with collisions of light and soon after heavy nuclei, at the AGS and SPS with energies up to 20 GeV per nucleon in the centre of mass frame, and later at the RHIC in Brookhaven, with energies of up to 200 GeV per nucleon. After more than a decade of experimental data analysis at SPS, CERN officially announced in 2000 “the compelling evidence for the formation of a new state of matter”.

Figure (1.7) shows the typical QGP phase diagram in the temperature vs. baryonic density, very similar to that first sketched by Cabibbo and Parisi in 1975. The phase transition region, predicted by QCD and based on a thermodynamical equation of state, is represented as a broad grey line roughly corresponding to the critical energy density of $\epsilon_c \sim 1 \text{ GeV/fm}^3$; it intercepts the temperature axis at the critical temperature $T_c$ of $\sim 170 \text{ MeV}$, in accor-

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for gluons. Each bosonic d.o.f. contributes by a factor $\frac{2}{3}$ to the energy density while each fermionic d.o.f. by a fraction $\frac{2}{3}$ of that value. For a two-flavour QCD, the number of effective degrees of freedom rises from $n_{dof} \sim 3$ of a typical a pion gas ($\pi^+,\pi^0,\pi^-$) to $n_{dof} = 37$. 

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Figure 1.7 – The QGP phase diagram.
dance with the discussed lattice calculations at zero baryo-chemical potential. Ordinary nuclear matter consists of nucleons of mass $\simeq 0.94 \text{ GeV}/c^2$ and has a density of $\simeq 0.17 \text{nucleons/fm}^3$ that implies an energy density of about $\simeq 0.16 \text{ GeV/fm}^3$. It is represented as a spot on the baryonic density axis which therefore sits well below energy density necessary for deconfinement.

During a nucleus-nucleus collision, differently from an elementary particle collision, the multiple interactions between the participant nucleons compel the produced energy quanta to rescatter of each other rather than to directly escape into the vacuum. As the energy of the colliding nuclei is increased, nuclei will more and more hit and penetrate into each other, leading to the formation of a dense "fireball" of highly excited nuclear matter which rapidly breaks up into nuclear fragments and mesons. Matter is compressed in this way, but lot of entropy is produced and hence it is also unavoidably heated up. The collision starts then at the cold nuclear matter spot at zero temperature in the phase diagram and then rapidly evolves towards higher temperatures and densities through early non-equilibrium stages. If energy is increased even further, nuclei start becoming more transparent, in the sense that a decreasing fraction of the beam energy and of the incoming baryons get stopped in the centre of mass of the system. In this transparency regime, which is mostly expected at extremely high energies like those of LHC, the baryonic content regions are displaced toward the beam rapidities and well separated from each other, while the mid-rapidity collision fireball gets more and more baryon-antibaryon symmetric [17]. Under these circumstances, the Lorentz-contracted nuclei are practically not slowed down at all and, especially for the most central collisions, nucleons are forced to perform many multiple interactions during a very short crossing time ($\sim \text{fm}/c$), resulting in a large quantity of energy to be released within a very small volume behind the nuclei. These are the best suited conditions to achieve the necessary energy density for the the phase transition to occur. If that is the case, then the hot and dense "fireball" of strongly interacting matter produced will eventually thermalize after some very short time into a quark-gluon plasma.

The early thermalized medium possesses a huge thermal pressure which acts against the surrounding vacuum and leads to a collective expansion which can be studied with hydrodynamic-based models. Expansion makes the fireball
cool down and lose energy, resulting in a sudden “bending” of the collision trajectory towards lower temperatures in the phase diagram. Eventually, the critical energy threshold $\epsilon_c$ is reached again and all partons are forced to convert into hadrons. Entropy density hence suffers a steep decrease whereas temperature remains approximately constant near $T_c$ which entails a huge increase of the fireball volume over larger time interval. During this time, the hadron gas particles still scatter of each other, and their chemical composition is constantly modified by inelastic collisions. When the rates of such processes become too small to keep up with the expansion, relative species abundances rest fixed and the so called chemical freeze-out is reached. Elastic collisions still keep occurring for a while, until eventually matter becomes so dilute that every strong process is out of range and all hadrons interactions stop. The momentum distribution now is fixed too and the thermal or kinetic freeze-out is reached. It is only at this moment that the so formed hadrons and their secondaries will be available for detection, and thus only from this cold and confined phase that any kind of information about the early hot and deconfined stage can be extracted.

1.3.3 Evidences of a new state of matter

At the beginning of the experimental programme, AGS and SPS where thought to be able to cross the transition line. The main question first sought by physicist was then how to get the “compelling proof” that a quark gluon plasma was produced and then, if that was the case, how to probe its properties with the available experimental tools. Alongside with colour deconfinement, an important theoretical feature, that is chiral symmetry restoration, that was actually predicted to occur at the phase transition. The observed spontaneous break of chiral symmetry is theoretically responsible, for example, of the manifest difference in masses between a nucleon ($m_N \simeq 940\text{ MeV}/c^2$) and its 100 times lighter bound quarks ($m_u \sim m_d \sim 2 - 6\text{ MeV}/c^2$). In the limit of their negligible masses, the light $u$ and $d$ quark are in facts theoretically described by a chirally symmetric QCD Lagrangian (that is to say, invariant under interchange of their left and right-handed field components), but the presence of a quark condensate,
formed through non-perturbative action of QCD gluons, breaks this symmetry group into the well-known isospin subgroup, so that the light quarks “dress up” of their soft interactions with the gluons increasing their mass value up to that of the effective constituent quarks. As in other physical situations, this spontaneous breaking is though expected to occur only under some critical temperature. When ordinary hadrons lose their identity and quarks become quasi-free in the deconfined phase at the QGP transition temperature, a partial restoration of chiral symmetry was computed and all quark effective masses were expected to return to their small “bare” value\textsuperscript{3}. The consequences would be a new central mass value or the with modification of light resonances (e.g.: \(\omega, \rho\), etc.), plus an increas of s quark production due to gluon energy loss. The presence of a deconfied medium, then would lead to a significant increase in the production of strange hadrons and it was indeed measured in recent experiments. This was considered as one of the first observable “signatures” of the formation of a QGP [18].

When hadronization occurs, in facts, the scattered partons recombine into hadrons in a statistical way and the production of baryons containing one or more strange (or heavier) quarks is enhanced due to the increase of free strange quarks. Partial chiral symmetry restoration, in facts, lowers the threshold for the production of a strange pair from twice the s quark constituent mass, \(\simeq 600\ \text{MeV}/c^2\), to twice the s quark bare mass, \(\simeq 300\ \text{MeV}/c^2\), leading to a significant enhancement of the observed strange baryons yield by a factor \(\epsilon_N\), with \(N\) being the baryon strangeness content. An enhancement hierarchy of the kind \(\epsilon_A < \epsilon_\Xi < \epsilon_\Omega\) for hyperions was then strongly expected, and indeed experimentally observed by WA97/NA57 experiments at the SPS [19][20]. Figure (1.8) shows the results of such measurements for both strange and anti-strange baryons, as function of centrality, in which an astonishing factor \(\simeq 10\) enhancement was observed for the \(\Omega\) baryon in the most central collisions.

Another historical signature that could provide this time evidences of the in-medium energy features of QGP had been sought and found by the physicists

\textsuperscript{3}The exact coincidence of the chiral and deconfined phase transition temperature is still under debate, however lattice simulations early showed how they were in any case very close to each other.
Figure 1.8 - Strange (left) and anti-strange (right) baryon production per event and per participant nucleon in Pb-Pb collisions, normalized to p-Be results, as function of the number of wounded nucleons, measuring centrality. Figure from [20] of the SPS heavy-ion programme when analysing $J/\psi$ and $\psi'$ meson yields. $J/\psi$ meson, to which the following chapter will be dedicated, as well as other heavy quarkonium states were indeed formerly appointed as very useful tools for probing hot medium characteristics and reveal the possible presence of de-confined state.

Due to their large mass, these bound states of heavy quarks require hard momentum transfers ($Q^2 \gg 1$ GeV/c) to occur. This not only means that their production can be more reliably computed by means of perturbative QCD, but also that, according to the uncertainty principle, their formation must happen at a very early time of the collision, of the order of $\tau_{\text{form}} \sim 0.1$ fm/c. Their early production plausibly makes these quark pairs subject to interferences while travelling in the dense and softer medium which, in the meanwhile, thermalizes, expands and cools down. A formed $J/\psi$ may then, in principle, carry information about all the evolution stages it expertised during the collision history.

Medium effects may interfere with the former quark pairs intention to hadronize, but there was actually another peculiar and long sought feature of QGP which
would have prevented them to bind together into a charmonium meson. As originally predicted by Matsui and Satz in 1986 [21], the colour-charge density environment of a QGP would have been high enough to screen the effective strong interaction between the two quarks of the $c\bar{c}$ pair, thus preventing the formation of their bound state. More details on this suppression, as well as on other more recently studied concurrent mechanisms, will be given in the following chapter. As for now, I will limit to state that this additional suppression with respect to “normal” medium effects, was indeed long studied and finally reported by the NA50 experiment, which measured the entity of such absorption in different ways, as function of the collision centrality, and compared it to the normal experimentally estimated absorption in nuclear medium. Figure

![Figure 1.9](image)

**Figure 1.9** – The $J/\psi$ to Drell-Yan ratio of cross-sections as function of the traversed nuclear matter $L$, for several collision systems, compared to (left) and divided by (right) the normal nuclear absorption pattern inferred from $pA$ collisions.

(1.9) shows a nice compilation of results from SPS experiments. The ratio of $J/\psi$ production to the Drell-Yann $q\bar{q} \rightarrow l^+l^-$ process cross section, used as a reference, is plotted as function of the length of traversed nuclear matter $L$ (a measure of centrality) and is compared to the “cold” nuclear absorption effects extrapolated from various analysed proton-nucleus collisions. These evidences were later considered as the other historical signature which allowed CERN to conclude that a new state of apparently non-confined matter, was produced in Pb–Pb collisions at these energies.

With the startup of RHIC in 2000, heavy-ion runs data started to be collected
at collider with energies up to 10 times those of fixed target SPS experiments. RHIC data early confirmed the overall picture emerging from the lower energy studies, but the higher initial temperatures and energy densities soon allowed more features of the collision processes and of QGP physics to be observed. Collective phenomena such as the hydrodynamic “flow” due to the hot medium expansion were found much more pronounced; moreover, the enhanced hard particles yields allowed more detailed studies on in-medium parton energy loss mechanisms. Evidence for a strong quenching of hard particles and jets traversing the hot medium created in central Au-Au collision was found in particular by the PHENIX and STAR experiments.

LHC is surely bringing heavy-ion collider physics into a new energy regime. This allows not only the opportunity to study, as already pointed out in section 1.3, a possibly “deeper” nuclear interaction, but also the chance of a better understanding of hot QCD matter. Higher initial temperatures should extend the life-time and the volume of the created hot medium, whereas the larger expected number gluons should favour momentum exchanges, reducing the time required for medium thermalization. Up to now, most recent ALICE measurements [5][23][24] in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV showed that initial temperatures up to 1.4 times those at RHIC and almost twice the critical value have been reached. Two times larger interaction volumes and 20% more extended life-times have been estimated, as well as a larger hydrodynamic flow and a 3 times higher measured energy density with respect to RHIC measurements. An increased nuclear transparency regime has also been observed, which implies a more “baryon-free” medium to be produced and a more suitable comparison with the mostly used zero baryo-chemical models predictions.

1.4 Study of Heavy-Ion Collisions

A fundamental aim of most heavy-ion collisions studies consists the extrapolation of all those quantitative aspect which can be considered “anomalous” with respect to what instead could be considered as “expected” from modellings based on known features of hadronic collisions. The task is not simple, not only because a solid knowledge of the elementary hadronic inter-
actions at the different energy scales must be provided, but also because a well-structured procedure to extrapolate the expected results to the complex scenario of a heavy-nucleus collision must be developed. Qualitatively speaking, in the absence of nuclear or medium effects, a nucleus-nucleus collision may be considered as a superposition of independent nucleon-nucleon collisions. Any modification with respect to what can be coherently inferred from this kind of picture may then be attributed to phenomena of physical interest, in the same way as what was done by SPS for finding evidences of the production of QGP.

At LHC energies, where the “bulk” of the produced hadrons in a heavy-ion collision is expected to come from hard scatterings between the nucleons, hence scaling with the number of binary nucleon collisions, a common observable to quantify the presence of anomalous physical effects is the nuclear modification factor, defined as:

$$R_{\text{AB}} = \frac{dN_{\text{AB}}/dp_T dy}{\langle N_{\text{coll}} \rangle \times dN_{\text{pp}}/dp_T dy}$$

(1.11)

that is to say as the ratio of the measured $p_T$ distribution $dN_{\text{AB}}/dp_T dy$ in nuclear A-B collisions, divided by the mean number of estimated binary nucleon–nucleon collisions $\langle N_{\text{coll}} \rangle$, to the $p_T$ distribution measured in p-p collisions, scaled to the same c.m.s. energy. This reveals to be useful choice since it also allows to reduce the systematic uncertainties, as the errors which are common to A-B and p-p data cancel out.

If no nuclear effects were present, the nuclear modification factor should be 1 for all those particle whose production is expected to scale with $N_{\text{coll}}$ (the so called binary scaling). On the contrary, an $R_{\text{AB}} \neq 1$ suggests something “out of the ordinary” has happened, and that further investigation must be performed to identify the causes which explain such differences. Effects which can modify the simple picture of a nucleon-nucleon superposition may have different origins and are usually divided in two independent classes:

- **Initial State Effects** account for all those effects due to interactions occurring during the propagation of the nuclei through each other, and which affect the hard cross sections in a way which depends on the size and energy of the colliding nuclei, but not on the medium produced after the collision. The understanding of these effects is crucial in order
to estimate the yield modifications coming from normal density “cold” nuclear matter (CNM).

- **Final State Effects** account instead for those effect induced by the interaction with the created medium, and which occurs at a much longer time-scale, when the high-energy nuclear debris are already far apart. The essentially depend on the proprieties (temperature, density,...) of the created medium and therefore are crucial in order to provide information on such proprieties.

A typical initial state effect can be seen in the modifications of the parton distribution functions in a nuclear target, relative to the nucleon case. This has proven to been especially relevant feature for high-energy collisions such as those at LHC, when deep probing scales (low Bjorken $x$) are involved. Being proportional to both $A$ and $\sqrt{s}$, the number of partons (mainly gluons) probed in these conditions becomes so high that they start overlapping in their phase space, eventually “saturating”, with their transverse size (induced by their intrinsic transverse momentum), the whole nucleus surface. Rather than a collection of separate partons, the gluons “seen” inside nuclei in these regimes can be described as an amorphous dense interacting system, often named *Colour Glass Condensate*, whose implication are still under debate and have attracted much attention in the recent times. Phase-space overlapping favours non-linear QCD interactions between gluons and other partons so that they will tend to merge together, summing up their fractional momenta ($g_{x_1}g_{x_2} \rightarrow g_{x_1+x_2}$) and consequently depleting the low $x$ region of their distributions. In a very simple picture, it is as if their crowding makes them “obsucure” each other so that their probed effective number is shadowed in experimental measurements. Such changes in nuclear PDFs have been parametrized by many authors and are being studied in the new energy domain by LHC experiments. Their modifications to particle production cross sections take place at the initial stages of the collision and must therefore be extrapolated in view of a compared study of nucleus-nucleus collisions.

A typical final state effect can instead be considered the energy loss of partons in the medium produced after a high-energy heavy-ion collision. This is surely of clear interest for the study of QGP related phenomena, but is
not peculiar of a deconfined medium. The mechanisms governing in-medium energy losses were first predicted by Bjorken [25] and have up to now been extensively studied. Gluon radiation, the QCD analogous to photon radiation in QED bremsstrahlung, appears to be the main source of energy loss in a strong interacting medium. It is predicted to be proportional to the square of the traversed medium length $L^2$ (contrary to the $\propto L$ proportionality of QED bremsstrahlung) as well as to be strongly dependent on the nature and on the properties of the medium, being, in the specific, much larger in case of deconfinement, where gluons are not confined into hadrons and thus not forced to carry only a small fraction of the total hadron momentum.

The main challenges thus lie in the tracing back of the observed results to the former sources of modification and, subsequently, in the discrimination of initial and final state interactions. Only if, having taken into account the correct initial state contributions, a successful explanation in terms of interaction with a “normal” medium can’t be found, one may start wondering about the formation of a QGP in the collision.

In virtue of what discussed above, I shall now conclude this chapter by explicating the fundamental role played in this context by nucleon-Nucleus collisions, that were the experimental environment of this thesis work study. Nucleon-Nucleus collisions can be considered, in effect, as the main linking bridge from the “known” elementary nucleon-nucleon collisions, in which no nuclear matter effects are present, and the complex Nucleus-Nucleus collisions, in which one must account not only for the ordinary nuclear matter effects, but also for the possible formation of a deconfined medium. Assuming that no such form of matter is produced in $p-A$ collisions, they then prove to be the necessary way to check out for the ordinary “cold” nuclear matter effects, so that they can reasonably be extrapolated to heavy-ion collision for later analyses.

That was what was done by SPS for the analysis of their results and, in a similar way, is what is even now done by present experiments. Again similarly as SPS $J/\psi$ measurements, the performed study reported in this work, can actually be considered as part of an analogous analysis on $J/\psi$ measurements in a “check-out” experiment consisting of proton-lead collisions, this time though, at the unprecedented c.m.s. energy of $\sqrt{s_{NN}} = 5.02$ TeV.
Chapter 2

$J/\psi$ Meson in Heavy-Ion Collisions

In the last chapter, I showed how one of the first historical evidences of the formation of an of deconfined matter was achieved through the study of heavy charmonia states. It should be easy then to understand why charmonia and, more generally, hard partons attracted much interest for the study of heavy-ion collisions. Being produced in the early stage of the collision in primary partonic scatterings with large momentum transfers, they not only offer a solid ground to probe several predictions of Quantum Chromodynamics, but also play a key-role for the disentangling of initial and final state modifications in nuclear collisions. Their initial production, in facts, can be considered as unaffected by the proprieties of the produced medium but only by the initial state effects on the target, whereas their propagation is able experience the full collision history, thus providing information on the final state effects in the strong medium. They are to all effects and purposes one of the most information-rich probes in high-energy physics.

This present thesis work will report an analysis performed on a data sample of $J/\psi$ candidates collected in $p – Pb$ collisions at $\sqrt{s_{NN}} = 5.02$ TeV by the ALICE experiment, and this chapter is hence meant to give a brief but proper description of the most significant features and experimental results in the field of charmonium physics in heavy-ion collisions, with particular reference given to the relevant case of the $J/\psi$ state.
After a short introduction of the different charmonia states, I will provide a description of the principal mechanisms and of the most-used proposed models describing $J/\psi$ production in heavy-ion collisions. An insight on the most recent conjectures regarding their role as probes of QGP as well as of other cold nuclear matter effects will be given in the third section, alongside with some of the most recent experimental findings. Last section will be finally devoted to the description of their use as indirect measure of beauty hadron production, which can be considered as the main purpose of the performed analysis.

2.1 The Charmonium Family

The attention devoted to heavy quarkonium states started with the so-called “November Revolution” in 1974 with the almost simultaneous discovery of the $J/\psi$ charmonium meson by Ting [26] and Richter [27] working groups, followed by the $\Upsilon$ bottomonium meson in 1977 by Leaderman group at Fermilab. The observation of the narrow $J/\psi$ mass peak at 3.09 GeV/$c^2$ furnished the direct proof of the predicted existence of the fourth quark $c$, awarding the Nobel prize to both Ting and Richter two years later, whereas the $\Upsilon$ peak confirmed the existence of the long predicted fifth quark $b$. Ever since their discovery, study of quarkonium states allowed significant progresses in the comprehension of the proprieties of strong interactions. Their relatively simple mass spectra and decay processes offered experimental ways to validate several QCD based models at different energy scales and, as already discussed, they are still nowadays considered as one of the most promising tools to probe the frontiers of QCD in heavy-ion collisions.

From a qualitative point of view, $J/\psi$ meson can be considered as a bound state of a $c\bar{c}$ pair, in a way very similar to the QED $e^+e^-$ positronium bound state. The collection of all those mesons made up of the same valence quark composition as $J/\psi$ constitute the so-called “charmonium family”. One of the reason which gave charmonium states such a primary role in the probing of QCD, was that, due to its heavy mass, it could be considered as a non-relativistic bound system, i.e. characterized by speeds substantially lower than $c$ in the c.m.s. of the $c\bar{c}$ pair. This early allowed a first-order description of its proprieties based
on the non-relativistic Schröedinger equation:

$$\frac{-1}{2\mu} \nabla^2 \psi(\vec{x}) + V(r)\psi(\vec{x}) = E\psi(\vec{x}) ,$$

(2.1)
in which $\mu = m_c/2 \sim 0.6\text{GeV}/c^2$ is the reduced mass of the quark pair, $r = |\vec{x}|$ the distance between the charm quarks, $\psi(\vec{x})$ the wave function describing the bound state in its centre of mass frame and $E$ the energy of the system.

An analogy with the $e^+e^-$ bound state quantum-mechanical description appears straightforward, accounting the substantial difference that now the QED Coulomb potential $V_C(r) = -\frac{\alpha}{r}$ is substituted by the strong potential function, expressed by the form:

$$V(r) = -\frac{4}{3} \frac{\alpha_s(r)}{r} + Kr$$

(2.2)
in which $\alpha_s$ represents the almost 30 times greater strong coupling constant and $K \sim 1 \text{ GeV}/\text{fm}$ represents the peculiar “string tension” of strong interaction, responsible of colour confinement due to the linear increase in field energy with increasing quark separation. One should know that $\alpha_s$ is not constant, but function of the quark separation distance $r$ and, hence, of their mass. By including some higher-order corrections to account for spin and relativistic effects, the energy level spectra can however be determined and the mass and coupling constants values estimated through direct comparison with experimental observations.

A scheme of charmonium states and their radiative transitions is reported in Figure 2.1, along with a table summarizing their fundamental proprieties in

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure2.1.png}
\caption{A scheme reporting the spectra and radiative transitions of charmonium family. From [28]}
\end{figure}
Figure 2.2 - Table illustrating the physical properties of the charmonium family members. The spectroscopic notation $n^{2S+1}L_J$ summarizes the state quantum numbers of spin $S$, orbital angular momentum $L$ and total angular momentum $J$, whereas the $J^{PC}$ notation specifies the state transformation properties with respect to parity $P$ and charge conjugation $C$ operators. From [28]

<table>
<thead>
<tr>
<th>Meson</th>
<th>$n^{2S+1}L_J$</th>
<th>$J^{PC}$</th>
<th>Mass (MeV)</th>
<th>Full Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_c(1S)$</td>
<td>$1S_0$</td>
<td>0'-'</td>
<td>2980.3±1.2</td>
<td>28.6±2.2 KeV</td>
</tr>
<tr>
<td>$J/\psi$</td>
<td>$1S_1$</td>
<td>1'-'</td>
<td>3096.916±0.011</td>
<td>92.9±2.8 KeV</td>
</tr>
<tr>
<td>$\chi_{c0}$</td>
<td>$1P_0$</td>
<td>0'+'</td>
<td>3414.75±0.31</td>
<td>10.5±0.08 MeV</td>
</tr>
<tr>
<td>$\chi_{c1}$</td>
<td>$1P_1$</td>
<td>1'+'</td>
<td>3510.66±0.07</td>
<td>0.88±0.05 MeV</td>
</tr>
<tr>
<td>$\chi_{c2}$</td>
<td>$1P_2$</td>
<td>2'+'</td>
<td>3556.20±0.09</td>
<td>1.95±0.13 MeV</td>
</tr>
<tr>
<td>$h_c$</td>
<td>$1P_0$</td>
<td>1'-'</td>
<td>3525.41±0.16</td>
<td>&lt; 1 MeV</td>
</tr>
<tr>
<td>$\eta_c(2S)$</td>
<td>$2S_0$</td>
<td>0'-'</td>
<td>3637±4</td>
<td>14±7 MeV</td>
</tr>
<tr>
<td>$\psi(2S)$</td>
<td>$2S_1$</td>
<td>1'-'</td>
<td>3686.094±0.04</td>
<td>589.188±0.028 MeV</td>
</tr>
</tbody>
</table>

Figure 2.2. Much like in atomic physics, the quantum numbers of charmonium states are enunciated in these Figures by the commonly used spectroscopic notations $n^{2S+1}L_J$ and $J^{PC}$, where $S$, $L$ and $J$ stand for spin, angular and total momentum operator eigenvalues, while the signs of $P$ and $C$ describe the state transformation properties under parity $P$ and charge conjugation $C$ operations. From tables, one may see that the $J/\psi$ meson coincides with the $S_1^3$ state of the charmonium bound state, whereas the ground state of the system is the singlet $S_0^1$ state occupied by the $\eta_c(1S)$ meson. The reason behind the earlier discovery of the $J/\psi$ state is that, contrarily to all other charmonium states which could be created only in hadronic colliders, $1^−−$ triplet states like $J/\psi$ or $\psi(2S)$ carry the same quantum numbers of a photon, and hence could be directly produced in the earlier $e^+e^−$ colliders by means of virtual photon production.

The detailed study of the radiative transitions, shown in Figure 2.1, among the various charmonium states allowed the experimental estimation of the spectrum physical constants as well as a more complete understanding of $q\bar{q}$ interactions. This is actually a surprising fact, since one may expect that a strong-produced particle should not exhibit such a pronounced electromagnetic decay yield. The main explanation I mean to point out for this is the important fact that $J/\psi$ mass value of $\sim 3.1$GeV/$c^2$ actually prohibits its decay into
“open charm”\(^1\) hadrons such as \(D\) or \(\bar{D}\), since these would require a higher threshold energy equal to the lowest charmed meson \((D)\) invariant mass of about \(\sim 3.8\text{GeV}/c^2\). In addition to this, is the fact that most other hadronic decays are suppressed since, for \(J^{PC}\) conservation reasons, they involve at least three gluons (or, less likely, two gluons plus one photon) in their decay process to occur; that means a rate suppression of the order of \(\sim \alpha_s^3\), which enough to let the electromagnetic processes compete, despite the small relative value of \(\alpha\), with the strong ones [29]. The cumulative electromagnetic channel decay rate of \(J/\psi\) rises up then to \(\sim 25\%\), which also implies a high branching ratio in the di-leptonic channel, of about \(\sim 12\%\). This is surely another experimental advantage for the study of charmonium production, being leptons efficiently resolved at hadronic colliders. The \(J/\psi \rightarrow e^+e^-\) and \(J/\psi \rightarrow \mu^+\mu^-\) channels are the most widely used (sometimes called “golden channels”) for \(J/\psi\) measurements since they provide a good signal over background \(S/B\) ratio. The reported analysis data sample was also obtained in such a way, consisting of \(J/\psi\) candidates reconstructed from their decay electromagnetic channel \(J/\psi \rightarrow e^+e^-\).

### 2.2 Charmonium Production in Hadronic Collisions

It has been already shown how the large mass of heavy quarkonium states qualifies them as precious tools for testing of QCD theory from nucleon-nucleon to heavy-ion collisions. Ever since their discovery though, the appearance of puzzling measurements has never stopped and has constantly led to new challenges for theorists. The mechanisms concerning \(J/\psi\) production could in facts be said to include QCD processes at different energy scales. A comprehensive theoretical description of the whole experimental results is still lacking, but much theoretical progress has surely been made.

The first stage of a \(J/\psi\), or more generally of a charmonium state production,

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\(^1\)Particles containing \(c\) or \(\bar{c}\) quarks and exhibiting a charm quantum number \(\neq 0\), like \(D\) and \(\bar{D}\) mesons, are called open charm particles, whereas \(c\bar{c}\) bound states, like charmonia, with charm quantum number \(= 0\), are called hidden charm particles.
consists in the production of a $c \bar{c}$ pair, that is, as said, a process well described by perturbative QCD in virtue of the large quark masses. The second stage is the evolution of the produced $c \bar{c}$ pair toward the proper $J/\psi$ or charmonium state, which is, on the contrary, a process characterized by soft interactions and hence out of the domain of perturbation theory, occurring on much longer time scales. It could be actually said that $J/\psi$ production lies halfway between the regimes of perturbative and non-perturbative QCD. The significant difference in the time scales between the two steps soon gave birth to models based on the so called “factorization approach”, i.e. on a factorization of the total production cross section in two terms relative to the two separate production stages. The first term is based on perturbative QCD calculations and concerns the hard production cross section of the heavy quark pair, while the second term involves the non-perturbative calculation of the probability for such a pair to evolve into a given charmonium state and is often described by means of effective models. I will provide in this section an overview of the fundamental mechanisms of charmonium hadroproduction in terms of the most employed models based on such approach.

### 2.2.1 Heavy quark pair production

The first step for producing charmonium state, that is a bound $c \bar{c}$ state, surely is that of creating a $c \bar{c}$ pair. In a hadronic collision, say a $p - p$ collision, this process may occur when a parton from the projectile nucleon interacts

![Figure 2.3](image-url) - Representation of heavy quark production from the hard scattering of two partons in a proton-proton collision.
with one from the target nucleon. This situation was actually already taken as example in section 1.2.3 for the general case of the production threshold of a heavy quark pair $Q\bar{Q}$ from a hard parton scattering. Recalling the simple formulas introduced there, it can be said that, in terms of their proton longitudinal momentum fractions $x_1$ and $x_2$, the total available energy in the hard scattering process for particle production is equal to the energy $\hat{s}$ of the two-partons in the centre of mass system and is given by

$$\hat{s} = (p_1 + p_2)^2 \approx x_1 x_2 s,$$  \hspace{1cm} (2.3)

with $s$, or equivalently $s_{NN}$ for the general case of ion collisions, being the c.m.s. energy of the colliding protons or nucleons $s = (P_1 + P_2)^2$. In order to produce a heavy quark $c\bar{c}$ or $b\bar{b}$ pair, it is of course necessary that the two partons c.m.s. energy $\hat{s}$ is at least as high as two times the constituent produced quark mass, that is for the charm case: $\sqrt{\hat{s}} \geq 2m_c \sim 2.4\text{GeV}/c^2$.

The elementary parton cross section terms for a $c\bar{c}$ pair production, as said, can then be calculated perturbatively and will in general depend on the momentum fractions and energy of the participant partons. Total cross section will be evaluated integrating over all the contributing partons as well as over all their possible momentums. These quantities are, as known, given by the Parton Distribution Functions $f_i(x, Q^2)$, which describe how the momentums of a parton $i$ (with $i$ being a quark $q$, anti-quark $\bar{q}$ or gluon $g$) is distributed inside a nucleon.

Figure 2.4 shows the most relevant leading order processes contributing to a $c\bar{c}$ production cross section in a hadronic collision. At high colliding energies when low Bjorjen $x$ values are probed (refer to section 1.2.3), the gluon PDF $f_g(x, Q^2)$ dominates, and most of the colliding nucleons momentum will hence be carried by gluons. Gluon fusion processes, namely the first two diagrams of Figure 2.4, are thus expected to be the dominant leading order contributions for $c\bar{c}$ production cross section at LHC energies.

### 2.2.2 Charmonium formation models

Once produced, the heavy charm quarks may in principle either combine with other light quarks to form open charm mesons ($D$ and $\bar{D}$) or bind with
each other to form a charmonium state. The $c\bar{c}$ pairs will generally be produced in a colour octet state, and will have to neutralize their colour in order to form a colourless charmonium meson. The mechanisms concerning colour neutralization and the resonance formation are though not yet fully understood, since they occur by interaction with the surrounding colour field and are presumably of non-perturbative nature [30].

One of the first historical models providing a good phenomenological description of charmonium production is the Color Evaporation Model (CEM), first proposed by Fritzsch [31] in 1977. It is based on the basic assumption that only a part of the total $c\bar{c}$ production cross section is relevant for the formation process, namely the so-called “sub-threshold cross section”, obtained by integrating the $c\bar{c}$ production cross section over energies below the threshold value for open charm mesons production $2m_D$. The probability of forming a specific quarkonium state is assumed to be independent of the colour and of the spin of the $c\bar{c}$ pair. Every charmonium state, in any colour configuration can be produced, and the $c\bar{c}$ pair is assumed to neutralize its color through the emission of many soft gluons, that is, by “color evaporation”, such that the final meson carries no information about the production process of the quark.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig2_4}
\caption{Lowest order Feynmann diagrams for the $c\bar{c}$ production in hadronic collision. In diagram (a) and (b) the quark pair is produced through gluon fusion processes, whereas in diagram (c) through quark-antiquark annihilation.}
\end{figure}
pair. According to CEM, the production cross section of any charmonium state is assumed to be just a fixed fraction of the sub-threshold charm cross section, independent from energy and which has to be determined empirically. This is a rather simple approach which nonetheless enjoyed a considerable phenomenological success for its simplicity and its experimentally well-supported qualitative predictions. When it comes to making more quantitative predictions, CEM though fails essentially because it can’t provide any prediction for the fractions of production cross sections, nor a consistent description of the colour neutralization process [30], a crucial aspect for charmonium production in nuclear collisions. Moreover, CEM assumes that the production rate of a quarkonium state should be independent of its spin state, so that it should always be produced unpolarized; fact which falls in strong disagreement with several experimental observations [32].

Another historical model, that is the Color Singlet Model (CSM), took place about in the same years [33]. It has been the first model which, contrarily to CEM, provided quantitative predictions on charmonium states production to be made in different colliding systems, from $e^+e^-$ to hadronic collisions. The name follows from its basic assumption that a specific charmonium state can be formed only if the $c\bar{c}$ pair is created in a colour-singlet state, with the same angular momentum quantum numbers as the charmonium. The heavy quarks creation process is treated perturbatively, and their non-perturbative binding is assumed to produce the bound states almost at rest, with vanishing quark relative momentum. Production rates are then related to the values of these bound state wave functions and their derivatives, evaluated at zero $c\bar{c}$ separation. CSM was actually believed at that time to be the most straightforward application of perturbative QCD to quarkonium production [34]. Despite the very different assumptions on which are founded, both CEM and CSM models enjoyed considerable phenomenological success through the 1980’s and into the 1990’s. At the present time though, even CSM can be excluded as a quantitative model of charmonium hadroproduction. Perhaps the most important evidence to date is the CDF analysis of direct\(^2\) $J/\psi$ and $\psi(2S)$ production

\(^2\)With the term “direct production”, one excludes the so-called “feed-down” contributions, i.e. productions via electromagnetic or strong decays from more massive states, such as higher-mass charmonium states.
at $\sqrt{s} = 1.8$ TeV in 1995, which revealed more than an order of magnitude discrepancy between the measured rates and the leading order CSM calculations. The situation at these high-energies seems to be improved by adding large higher order correction terms [35][36], but a fully consistent predictive picture still remains out of sight.

The experimental puzzles prompted the introduction of new ideas and soon gave rise, in 1995, to the effective theory of Non-Relativistic Quantum Chromodynamics [38] (NRQCD), which encompassed CSM going beyond most of its limitations. At present, NRQCD appears to be the most theoretically studied factorization approach to heavy-quarkonium production, as well as the most successful phenomenologically. The essential trait is that, given the low heavy-quark velocities $v$ involved, NRQCD studies charmonium production with a non-relativistic approach, so that the factorization formula can be expressed by a double expansion in powers of $\alpha_s$ and powers of $v$. While the short-range quark pair production is still, as in CSM, treated perturbatively, the long-range non-perturbative evolution of the pairs into charmonium states is expressed in terms of matrix elements of NRQCD operators which can be characterized with respect to their scaling with $v$. The inclusive cross section depends both of these stages and, in phenomenological applications, truncated at some fixed order in $v$, so that and only a few matrix elements typically enter into the phenomenology. This time, not only colour-singlet states are taken into account in the NRQCD approach. If one considers only colour-singlet contribution in the expansion at leading order of $v$, then the CSM is obtained. The full inclusion of colour-octet contributions, instead, leads to the often called Colour Octet Model (COM), according to which, the coloured $c\bar{c}$ pair evolves towards the colourless resonance state by combining and subsequently absorbing, after an average “relaxation” time $\tau_s \simeq 0.25$ fm/c [30], a soft collinear gluon.

Although the application of NRQCD factorization to heavy-quarkonium production processes has had many successes, there remain a number of discrepancies between its predictions and experimental measurements, especially for what concerns $J/\psi$ polarization and photo-production measurements [37]. It can be said that despite recent theoretical advances, a clear picture of the mechanisms at work in quarkonium hadroproduction is still lacking.
2.3 Charmonium Production and Absorption in Nuclear Medium

The picture of charmonium production traced in the previous section for elementary hadronic collisions is surely expected to be very different in proton-Nucleus or Nucleus-Nucleus collisions. The presence of deconfined, or even nuclear matter can affect $J/\psi$ production during its entire evolution as well as in different ways. In particular, production can be affected either at the very initial stage of the collision, due to the presence of other nucleons in the target nucleus which can alter the perturbative $c\bar{c}$ pair production process, or in the later stages, e.g. due to the produced pair interactions with the nuclear medium. If on the one hand several different phenomena are expected to act on charmonium formation in nuclear collisions, on the other hand, these make charmonium states the most information-rich probes to investigate them. For this reason, a wealth of experimental observations have long been collected and studied in considerable detail, though, as I will point out in the following brief overview, the situation still remains puzzling and the many developed theoretical approaches seems unable to provide a global picture. Out of this vast research scenario, I will trace the most commonly proposed effects expected to play a consistent role in the production of $J/\psi$ and other charmonium states, either in cold or in hot and deconfined nuclear matter, including at the same time some significant as well as more recent experimental findings.

2.3.1 Charmonia in proton-nucleus collisions

Since the most important goal for charmonium studies in heavy-ion collisions is eventually the investigation of the effects which a secondary hot produced medium has on its production, it is essential to account, first of all, for any effects due to the initial nuclear medium in a correct way. In section 1.3.3, I reported as one of the long-considered historical evidences of the formation of a deconfined medium the observation, made by NA50 experiment, of an anomalous $J/\psi$ suppression in Pb-Pb collisions at $\sqrt{s_{NN}} = 158$ GeV. In order to define what was to be called as an “anomaly”, the measurements accounted of the estimated “normal” modification effects due to
the presence of a dense nuclear target. Normally, all these kind of estimates are based on observations previously made in nucleon-nucleus collision experiments, where the production of a hot deconfined medium is not expected. This is actually the most natural way to disentangle the two kinds of contributions and, at the same time, to understand more about the complex phenomena interplaying in \textit{Cold Nuclear Matter} (CNM). How one defines CNM effects is thus not only an important, but a crucial preliminary feature which has to be considered before any further considerations can be made. Assuming the role and weight of each nuclear effect could be correctly quantified, further difficulties arise in how each of the effects estimated in a given nucleon-nucleus collision can be correctly extrapolated to a much more complex nucleus-nucleus collision, possibly at different energies or under different kinematic conditions. Such an extrapolation revealed to be not as straightforward as one may think.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.5}
\caption{Illustration for the exponential attenuation of charmonium in a nuclear target of density $\rho_A$, assuming an average absorption cross section $\sigma_{\text{abs}}$.}
\end{figure}

When considering the case of $J/\psi$ or, more generally, of charmonium production in a $p - A$ collision, the simplest assumption one can make is that a $J/\psi$ produced in the nuclear medium is suppressed, with respect to the $p - p$ case, with a constant average absorption cross section $\sigma_{\text{abs}}$, which causes its exponential attenuation over the average traversed path length $L$ in the nuclear target (Figure 2.5). In the frame of Glauber formalism, the nuclear modification factor $R_{pA}$ would then take the form [39]:

\begin{equation}
R_{pA} = \frac{1}{A\sigma_{\text{abs}}} \int d^2b \left( 1 - e^{-\sigma_{\text{abs}} T_A(b)} \right),
\end{equation}
where the impact parameter dependent function \( T_A(b) = \int -\infty \to +\infty dz \rho_A(b, z) \)
accounts for the traversed nuclear target thickness.
Nuclear effects underlying the SPS anomalous \( J/\psi \) suppression where actually evaluated in such a way, by extracting \( \sigma_{abs} \) from the various \( p-A \) data at 400/450 GeV and subsequently extrapolating it at the lower energy of \( \sqrt{s_{NN}} = 158 \) GeV, assuming in both cases the scaling with \( L \) and imposing it to be energy independent.

On a more detailed view, one should note however that \( \sigma_{abs} \) is actually an effective quantity, since it represents the overall amount of cold nuclear matter effects reducing the \( J/\psi \) yield, but doesn’t allow to distinguish the different contributions playing a role in this reduction. These contributions, as anticipated, may come into play during all the phases of the charmed dipole evolution, since the initial stage of \( c\bar{c} \) pair production, which may be subject to suppression or enhancement accordingly to the nuclear modification of the parton distribution functions, or even before if one considers, for example, energy losses of the former parton traversing the nucleus before the hard scattering. It is clear that a precise study of these processes is needed in order to validate any assumptions made when extrapolating CNM effects from \( p-A \) to \( A-A \) collisions, especially when different energies or kinematic domain are involved.

As a starting point, one may consider as a fundamental initial state effect, which was also neglected in the former NA50 data, the quoted modification of PDFs in a nuclear medium. As discussed in section 1.4, the simple presence of other nucleons in a nucleus is enough to modify, with respect to the nucleon case, the initial state distribution of the partons which enter in the perturbative creation of the \( c\bar{c} \) pair. Such modifications are usually determined by means of deep inelastic scattering experiments off nuclei, and have by now long been studied from various working groups, which offer different parametrizations of the modification functions. Among them, I will mention the first one made by the EKS98 group, in 1998, and the some of the most used later ones such as nDSg, HKN07 and EPS09. While quark and anti-quark modifications are relatively well-measured, gluon density modifications rely on several constraints since are not directly measured. Consequent wide variations in the
shadowing effects at very low Bjorken $x$ and in the anti-shadowing region at $x \sim 0.1$ are common to all parametrizations, and further explain the discussed need for more direct measurements. Figure 2.6 shows a compilation of the most used nuclear PDF modification functions, relative to the nucleon case, for both quark and gluon density distributions.

Another important effect which may characterize the initial state production of charmonia, is the fact that nuclear targets, either in $p - A$ or in $A - A$ collisions, modify the transverse momentum distribution of produced particles. It is long known phenomenon of parton transverse momentum broadening, sometimes referred as Cronin Effect. It was first observed in proton-nucleus collisions in the late 70’s, and consists in a hardening of transverse momentum spectra, relative to proton-proton collisions, which sets in at medium-high $p_T \sim 1 - 2$ GeV/$c$, disappears at much larger $p_T$’s, and to which corresponds a depletion at low transverse momenta. The main effect explaining this lies in the momentum broadening due to the multiple scatterings of partons from the proton off partons from the nucleus. In a simple quantitative description, it is usually described as if, in a random walk analysis, incident partons suffer random “kicks” on each collision, which shift their momenta from lower to higher values, and hereby cause the observed respective depletion and enhancement. This is a well-established effect which has been observed also in $J/\psi$ measurements, although a quantitative agreement, based on QCD, for
all its features (such as, for instance, the flavor dependence) is still lacking [41].

Accounted the production yield modifications due to the above stated initial state effects, which are intrinsic to the nuclear target, one has to consider all those effects concerning the break-up, i.e. the absorption, of the quarkonium state as it passes through the nucleus. In doing this, one has to remember that such absorption can be suffered in the pre-resonance as well as in the resonance stage, due to the successive interactions with the target nucleons. Since these steps are very different in nature, much attention should therefore be paid in evaluating what are their characteristic time scales within the colliding system. Assuming that the charmonium is produced at a random point inside the nucleus, one should hence know in which stage of its evolution it will be along its path through nuclear matter.

As discussed in the previous section, if production passes through a colour-octet state, it requires a time $\tau_8$ for the initially produced $c\bar{c}$ pair to neutralize its colour. Moreover, according to the uncertainty relation, a further time $\tau_f$ is also required for the colourless dipole to acquire a fixed mass and form the actual physical $J/\psi$ ground state. During this time the colour-singlet $c\bar{c}$ dipole, initially produced with a starting small separation $r_{c\bar{c}} \sim 1/m_c \sim 0.1$ fm, is expected to evolve up to $J/\psi$ mean size $r_{J/\psi} \sim 0.5$ fm, as schematised in Figure 2.7.

These times are very small in the dipole rest frame, of about $\tau_f \lesssim \tau_8 \sim 0.2$ fm/c [40], but may become large in the nuclear target frame due to Lorentz time dilatation. The crucial quantity for accounting this is the momentum or, equivalently, the energy $E_{c\bar{c}}$ of the charmonium dipole state as measured in the nuclear target rest frame. At low energies such as those of SPS and below,

![Figure 2.7](image.png)

*Figure 2.7 – Schematisation of the evolution of a $c\bar{c}$ dipole propagating through a medium.*
with corresponding $E_{c\bar{c}} \lesssim 25$ GeV, the overall formation time is shorter than the average nucleon spacing in a nucleus of $\sim 2$ fm so that one can treat the formation process as instantaneous and consider a fully formed $J/\psi$ to travel the nucleus, with an absorption cross section in (2.4) which is then given by the inelastic charmonium-nucleon cross section $\sigma_{in}^{J/\psi N}$.

With increasing energies, the characteristic formation time scales start overlapping with the nucleus radius $R_A$, and it becomes relevant whether the object traversing the nucleus is a precursor color-octet state or a formed colour-singlet dipole state. In the latter case, one may realize that absorption would be different accordingly to the actual dipole separation (Figure 2.7). A fully-formed $J/\psi$ state has a greater transverse size $r_{J/\psi} \sim 0.5$ and hence may show a greater absorption cross section, e.g. $\sigma_{abs} \propto r_{J/\psi}^2$, with respect to the initially small separated dipole. Also, as a consequence, if part of the passage is carried out as physical resonance, higher excited states should lead to even larger absorption cross-sections than the much smaller ground state $J/\psi$.

In this picture, the effective absorption cross section should drop with increasing $J/\psi$ energy, as the nucleus should get more and more “transparent” to the charm dipole. This is because the higher the energy, the more the $c\bar{c}$ dipole is “frozen” to its initial small size. Data from fixed target experiments, reported in the left panel of Figure 2.8, indeed soon seemed to confirm this trend as function of charmonium energy $E_{c\bar{c}}$ in the target nucleus frame.

If it is the case of a colour-octet object to be produced in the initial stage, then one could point out that, for sufficiently high energies and dilated neutralization times, part of the charmonium passage through the nucleus could be carried as a pre-resonance state, which could then be dissociated through interaction with nuclear matter. In this case, it is assumed to immediately interact with a large, finite dissociation cross section since it is a coloured object. Furthermore, it has been often argued that, in this time, all the precursor quarkonium states will be indistinguishable, so that the absorption effects at these high energies should be the same for all the states [40]. Experimentally, although the difference between the effective $A$ dependence of $J/\psi$ and $\psi(2S)$ states seems to decrease with increasing beam energy, the collection of fixed target analyses results collected over the range of energies from 400 to 800 GeV shows that their absorption cross section are not identical, as the ba-
sic color-octet absorption mechanism would suggest [37], and that the $\psi(2S)$ state gets systematically more suppressed [42]. The overall behaviour of $J/\psi$ absorption at mid-rapidities deduced from these experiments, including the more recent HERA and RHIC data up to $\sqrt{s_{NN}} = 200$ GeV, confirms instead the stated decrease energy, regardless of the chosen parametrization of nPDFs, as is clearly shown from the compilation of results in the right panel of Figure 2.8.

One last crucial effect that I am to report is the astonishing kinematic dependence over rapidity of nuclear charmonium absorption which has been observed throughout a whole variety of colliding systems either in $p - A$ or $A - A$ collisions, as well as in other lighter hadrons. It manifests as a drastic increase of nuclear effects towards large c.m.s. rapidities. Very commonly the effect is reported as a function of the Feynmann variable $x_F = p_{LJ/\psi} / p_{LJ/\psi}^{max}$, which represents, in inclusive experiments, the longitudinal momentum fraction of the

![Figure 2.8](image)

**Figure 2.8** (left): Effective absorption cross section $\sigma_{abs}$ as function of charmonium energy $E_{cc}$in the nuclear rest frame. The curves represent theoretical expectations [39] for an $c\bar{c}$ expanding dipole, for different starting sizes. (right): C.m.s. energy dependence of charmonium absorption cross section $\sigma_{abs}$ at mid-rapidity. The solid line is a power law approximation which makes use of EKS98 parametrization, the band indicates uncertainties while the dashed curve shows an exponential fit for comparison. Data from left panel are shown along with data from NA3, HERA-B and PHENIX. The vertical dotted line indicates the energy of the Pb+Pb and In+In collisions at the CERN SPS. Figure from [37]
produced particle relative to the maximum longitudinal momentum $p_{L \text{ max}}^{J/\psi}$ achievable under the experiment energy and kinematic conditions. The compilation of fixed target experiment results, reported in Figure 2.9 shows a clear systematic increase of absorption effects from low to high values of $x_F$, as well as a similar apparent trend from high to low system energies. Though the phenomenon is most known, it is striking that there is no consensus yet on the origin of such dependence. Since large Feynmann $x_F$'s correspond to low Bjorken $x$'s of the target gluons, one may feel tempted, for example, to relate such effect with the increase of nuclear suppression due to shadowing effects in the target nucleus, but this is not the case, since the effect doesn’t seem to scale with $x$. Despite many theoretical attempts actually not even a common scaling variable for the different effects has been found.

One natural explanation, among the different proposals, can be given in terms energy-dissipation related mechanisms which affect the projectile hadron and its debris on their way through the nucleus and which result in a deficit of energy for nuclear processes at large $x_F$ [39]. Results from high-energy colliders as function of rapidity are reported in Figure 2.10 and compared to some theoretical formulations. Either RHIC data [44] or the most recent ALICE data [45], taken at large rapidities, show the same trend as other above stated

**Figure 2.9** – Compilation of $J/\psi$ absorption cross sections $\sigma_{\text{abs}}$ as a function of $x_F$ with no additional cold-matter effects included. Figure from [43].
fixed target experiments.

Figure 2.10 – left: RHIC data [44] for $J/\psi$ nuclear modification factor in $d – Au$ collisions at 800 GeV/c, at large rapidities. The curves shows theoretical prediction for an energy-loss based model [46], for various assumptions regarding the nuclear modifications of gluon distributions in nuclei. right: The most recent LHC measurements [45] of $J/\psi$ nuclear modification factor from $p – Pb$ collisions at $\sqrt{s_{NN}} = 5.02$ TeV. Some different theoretical prediction are shown.

2.3.2 Charmonia in Nucleus-Nucleus collisions

Accounted for for all the modifications due to interactions in normal cold nuclear matter, one may wonder about probing the further effects that the hot strong interacting medium produced after a heavy-ion nucleus-nucleus has on charmonium production. This time, even at RHIC and LHC energies, most of the $J/\psi$’s will be produced with relatively low transverse momenta in the rest frame of the medium. The characteristic formation times are shorter than the mean thermalization time and much shorter than the mean traversed path in the medium ($\sim 5 – 10$ fm), so that in this case it is a fully formed $J/\psi$ one has to take into account for in-medium processes [39]. The presence of a hot medium alone, boosts the mechanisms of energy loss discussed in section 1.4 since they are expected to become more consistent with increasing temperature, but if the hot medium thermalizes into a QGP, then many out of the ordinary effects are expected to be found.
As already introduced, charmonia have long been labeled as “signatures” of QGP, that is, capable of bringing some evidence of the formation of a deconfined medium. The main reason for this, which was introduced in section 1.3, lies in their peculiar expected suppression due to the colour-screening effect originally predicted from Matsui and Satz [21] in 1986. To look at this with more detail, one can start by considering an analogous effect, borrowed from atomic physics, that is that of electric charge screening in atomic matter.

In a dense atomic medium, due to the overlapping of the atomic orbits, the effective charge of each nucleus gets partially screened by the electronic orbits of other nearby atoms which produce a high density negative charge around the positively charged nucleus. The phenomenon is referred as Debye screening. As a consequence, the Coulomb potential $V(r)$ between two electric charges becomes:

$$V(r) = \frac{e}{r} \rightarrow \frac{e}{r} \cdot e^{-\frac{r}{r_D}}, \quad (2.5)$$

where $r_D$ is the so-called Debye radius, which characterizes the new effective electrostatic interaction between the charges. In conductors, $r_D$ is typically smaller than the distance between the nuclei and their outermost electrons, which hence feel a smaller electric field and behave as almost free particles inside the material. In an insulator instead, $r_D$ is larger than the radius of the atom, so that the electrons are confined. The relevant feature is that Debye radius $r_D$ is inversely proportional to the matter density $\rho$ and to the temperature $T$ of the system, therefore with increasing $\rho$ or $T$, $r_D$ becomes smaller and smaller until the insulator material eventually undergoes a phase transition to a conductor, which takes the name of Mott transition.

In a very similar picture, a medium characterized by a very high density of colour charge should be affected by an analogous “colour screening effect”, so that the colour inter-quark potential, introduced in eq. (2.2) gets attenuated as

$$V(r) \sim -\frac{\alpha_s}{r} + K \cdot r \rightarrow \frac{-\alpha_s}{r} \cdot e^{-\frac{r}{r_D}} + K \cdot r_D (1 - e^{-\frac{r}{r_D}}). \quad (2.6)$$

With increasing temperature or colour charge density, the binding force which holds the quarks inside hadrons vanishes and deconfinement eventually occurs. The Mott transition in an atomic medium can thus in principle be considered as an analogous for the phase transition to a “colour conductive” QGP in
QCD.

When looking at charmonium states produced in a heavy-ion collision, this brings up some interesting consequences. As temperature increases, in facts, the colour Debye radius $r_D$ will eventually become smaller than the binding radius of the $c\bar{c}$ pair, preventing the formation of a charmonium state. Since the different charmonium states are characterized by different binding energies and consequently by different binding radii, one can then foresee that, with increasing temperature, the progressive decrease of $r_D$ cause a sequential “melting” of the different states, starting from the larger and less bound ones up to the smaller and more tightly bound $J/\psi$. The melting will occur then at specific threshold temperatures, making charmonium and quarkonium states in general a unique “thermometer” for the QGP temperature (Figure 2.11, left). Experimentally, this fact translates in what perhaps the most characteristic feature predicted for charmonium production in nuclear collisions. The $J/\psi$’s actually measured in inclusive experiments present in facts contribution coming from the feed-down from higher excited states. Only about 60% of the measured $J/\psi$’s measured in hadron-hadron collisions are directly produced $1S$ charmonium states, while about 30% come from the decay $\chi_c(1P)$ states, and the remaining 10% from $\psi'(2S)$ [47]. This means that, in virtue of QGP colour screening features, the expected inclusive $J/\psi$ yield will present a pe-

![Figure 2.11](left): A cartoon representing the quarkonium states as thermometer of quark-gluon plasma. As temperature, or energy density, increase, the different states will sequentially “melt”. (right): Scheme of sequential melting of feed-down contributions to $J/\psi$ production due to colour screening.
Figure 2.12 – Estimated \( J/\psi \) suppression relative to the cold nuclear matter modification ratio \( R_{AA}(CNM) \), measured by SPS and RHIC experiments as function of charged multiplicity at mid-rapidity \( dN_{ch}/d\eta|_{\eta}=0 \). From [37].

culinar stepwise suppression pattern as function of energy density. Figure 2.11 schematically illustrates the expectation for the \( J/\psi \) survival probability in these conditions, normalized to its normal nuclear suppression value.

Figure 2.12 shows the most relevant experimental results from SPS and RHIC heavy-ion experiments, along with their systematic uncertainties due to the evaluation of cold matter effects, as function of energy density, evaluated by means of particle multiplicity at mid-rapidity. A distinct stepwise trend is invaluable because of the large uncertainties, but the anomalous suppression reported by the different systems seems to agree qualitatively with the thermal dissociation scenario.

At this point, I shall now remark an inexplicit aspect on which all the argumentations I reported until now concerning QGP charmonium suppression are based on. That is the crucial assumption that charmonia, once dissociated, cannot be recreated during the thermal evolution of a QGP. This inexplicit assumption actually relies on the fact that the abundance of charm quarks in an equilibrium QGP should be far too low to allow it. Compared to that of light quarks, the thermal production rate of charm quarks in an equilibrated QGP can in facts be estimated to be proportional to a factor of about \( \sim e^{-m_c/T_c} \sim 6 \times 10^{-4} \) [34]. The initial abundance of charm quarks in a heavy-
ion collision, however, is actually expected to be much higher than this value. That is because, contrarily to light quarks, whose production rate is proportional the number of participant nucleons $N_{\text{part}}$, charm quarks are produced through hard scattering processes, which are proportional to the number of binary collisions $N_{\text{coll}}$, much higher than $N_{\text{part}}$. Relative to a $p - p$ collision, in which $N_{\text{coll}} = 1$, a heavy-ion collision will thus present an "excess" of charm pairs coming from the multiple nucleons interactions, growing with the centrality of the collision. This initial excess of heavy-quark abundance ratio is furthermore expected to increase with energy, since the charm pair production cross section grows faster with energy than that for light hadron production. These kind of considerations led to introduce an alternative approach, which has been proposed [48] to account for the available data, drastically opposed to the above discussed suppression, and which accounts the possibility of a regeneration scenario for $J/\psi$ production in high-energy heavy-ion collisions. The main concept is that, if one assumes that the initial non-thermal charm oversaturation is preserved throughout the subsequent hot medium evolution, it is possible for a charm $c$ quark from a given nucleon-nucleon collision to combine with another charm $\bar{c}$ quark from a different collision to recreate a $J/\psi$ at the hadronization stage, in a sort of statistical way. For what has been said before, then this kind of statistical recombination of "off-diagonal" $c\bar{c}$ pairs should become more and more likely with increasing energy, up to the point

Figure 2.13 – (left): Schematisation of the statistical recombination scenario, at RHIC and LHC energies. (right): $J/\psi$ survival probability, normalized to normal nuclear absorption effects, including the contributions due to sequential melting from colour screening and statistical regeneration.
where it would compensate the effects of $J/\psi$ dissociation, and eventually lead, at much higher energies, to an overall $J/\psi$ enhancement relative to the rates scaled from $p - p$ results. Figure 2.13 summarizes these concepts, showing the two very different discussed scenarios in terms of the overall $J/\psi$ survival probability. The predictions for LHC energies are therefore ideally opposite extremes, which fortunately lead to hope for an eventual resolution. So far the latest results from $Pb - Pb$ collisions at $\sqrt{s_{NN}} = 2.76$ TeV, reported in Figure 2.14, seem to show a rather different trend if compared to the lower energy RHIC data, apparently in qualitative agreement with a regeneration scenario.

2.4 $J/\psi$ and Measurement of $B$ Hadron Production

Apart from the hidden-flavour bound quarkonium states, open-flavour heavy quark hadrons represent another source of valuable information for the investigation of medium effects in heavy-ion collision. Given the unprecedented large production cross section of $b\bar{b}$ pairs offered by LHC energy domain, much effort has been recently devoted to the identification and measurement of the very heavy beauty-flavoured hadrons.
B mesons offer a much higher mass scale than the lighter D charmed mesons and their comparative study may thus reveal new aspects on heavy quark production and propagation in the nuclear medium. To state an example, one of the most commonly investigated features is the predicted hierarchy in energy loss they are expected to manifest when travelling in a strong medium. In very simple terms, when computing the QCD energy loss for a parton travelling a dense medium, the available phase space for gluon radiation happens to depend on the mass of the travelling parton. As a consequence of this effect, which is known as dead cone effect, heavier quarks are expected to radiate less gluons than lighter quarks, hence exhibiting less energy loss at the final stage of a nuclear collision. Such a behaviour should become significant in heavy-ion collisions, where a hot strong interacting medium is expected to be produced, and should translate into an hierarchy of the measured mesons nuclear modification factors in the form $R_{AA}(B) > R_{AA}(D) > R_{AA}(K,\pi)$, corresponding to the energy loss scale $\Delta E(u,d,s) > \Delta E(c) > \Delta E(b)$. So far, data collected by ALICE experiment for D and π mesons $R_{AA}$ at low $p_T$ seem show a slight agreement with the above stated trend, but measurements are still not conclusive due to the high statistical uncertainties. Heavier B hadrons are expected to prove an even accentuated effect, so their measurement proves to be very useful for further investigations.

Charmonia happen to play again another important role in this context as they provide a privileged source of measurement for beauty hadron production. One must consider, in facts, that apart from the stated feed-down contributions from higher excited charmonium states, $J/\psi$ production in hadronic collision may occur also through the decay of beauty hadrons, in processes like $H_B \rightarrow J/\psi + X$. The inclusive branching ratio for this kind of processes, evaluated on the basis of LEP, Tevatron and SpS results, has been estimated to be of $(1.16 \pm 0.10)\%$ [50], which means that at LHC energies, there will be a significant and measurable component of the inclusive $J/\psi$ yield resulting from beauty hadrons decays.

Figure 2.15 gives a quantitative insight of the topic. The fraction

$$f_B = \frac{B \rightarrow J/\psi + X}{anyinclusiveJ/\psi},$$
measured from the different LHC experiments, is reported as function of $J/\psi$ transverse momentum, and compared to older high-statistics CDF results [34]. It is remarkable how, despite the different energy and kinematic regimes, the various experiments measured the same common increasing trend as function of $p_T$, proving also that beauty decay becomes the most significant component of the yield at very high transverse momenta.

The indirect measurements of $B$ hadrons by means of analyses on $J/\psi$ inclusive production arise then as valuable complement to open-flavour measurements, capable of providing more precise information on beauty production cross section and nuclear effects.

As far as now, I have introduced all the necessary concepts for the explication of the performed analysis and given it a proper contextualization within the rich framework of heavy-ion physics research experiments. The purpose of the work should then appear very clear. The aim has been to extract the $f_B$ fraction from a data sample of inclusive $J/\psi$ candidates reconstructed from their electromagnetic decay channel $J/\psi \to e^+e^-$ in a collection of minimum-bias $p - Pb$ collisions at the c.m.s. energy per nucleon pair of $\sqrt{s_{NN}} = 5.02$ TeV in the central rapidity region at the ALICE experiment at LHC.
Chapter 3

The ALICE Experiment at LHC

ALICE (A Large Ion Collider Experiment) is the dedicated heavy ion experiment at the Large Hadron Collider. Its detector structure has been built by a collaboration of over a thousand physicists and engineers from almost 30 different countries, and has been specifically driven for the study of the expected physical conditions in the high-energy heavy-ion Pb − Pb collisions at LHC.

This short chapter will be devoted to the description of ALICE experiment and is therefore meant to provide some specific insights on the experimental background underlying the analysis which will be reported in the next chapter. After a short overview on ALICE performances, a general description of the whole detectors layout will be presented in section 3.1. Emphasis will then be given on the role ITS and TPC detectors in section 3.2 and 3.3 respectively, along with some highlight on the secondary vertex resolution and TPC particle identification, particularly relevant for the reported analysis data acquisition and selection.

3.1 Detectors Layout

The requirement of the capability to track and identify particles in an environment with a charged particle multiplicity which was originally predicted...
to reach the order of 8000\textsuperscript{1} per unit rapidity at mid-rapidity, led to a unique design for ALICE detector, with a very different optimization with respect to other \( p - p \) dedicated experiments at LHC. One of its main characteristic is the exceptional capability of tracking and identifying particles in the central rapidity region down to very low values of transverse momenta, often inaccessible to other LHC experiments, which is achieved thanks to the combined presence of fairly low magnetic fields, with a maximum intensity of 0.5 T, and of a maximally reduced material budget, to minimize multiple scattering effects.

Transverse momentum measurements cover a range of at least three orders of magnitude, from about \( \sim 100 \text{ MeV/c} \) up to values higher than \( \sim 100 \text{ GeV/c} \). In order to study QCD and quark de-confinement under extreme energy conditions, and over such a wide momentum range, particle identification (PID) is then essential. A key design consideration of ALICE is its capability to identify particles by making use of practically all the known PID techniques: specific energy loss \( \frac{dE}{dx} \) measurements, time of flight measurements, Cherenkov and transition radiation, electromagnetic calorimetry and topological reconstruction of particle decays.

A schematic view of the general ALICE layout is shown in Figure 3.1. It consists of a main central barrel detector (\(|\eta| < 0.9\)) covering the full azimuth, where hadrons, electrons and photons are measured, and of a forward muon arm (\(2.4 < \eta < 4\)). Other smaller detectors are also displaced in the forward rapidity region and are employed for global event characterizations, such as multiplicity and centrality measurements.

ALICE global reference frame is defined so that it has the \( z \) axis parallel to the beam direction and pointing towards the muon arm, and \( x \) and \( y \) axes in the plane transverse to the beam direction.

The central barrel is embedded in a large solenoidal magnet with a rather weak field of intensity < 0.5 T, parallel to \( z \), and it consists of the Inner

\textsuperscript{1}First estimates of the density of charged particles \( \frac{dn}{d\eta} \) at mid-rapidities for central \( \text{Pb-Pb} \) collisions at \( \sqrt{s_{NN}} = 5.5 \text{ TeV} \) covered the range from 2000 to 8000. The subsequent RHIC measurements lowered this value down to 4000, according to which the detector performances were optimized. Current extrapolations predict a multiplicity \( \frac{dn}{d\eta} \simeq 2000 \) for central collisions at \( \sqrt{s_{NN}} = 5.5 \text{ TeV} \), still much lower than expectations.
Figure 3.1 – Layout of the ALICE detector. The 18 detectors of the experiment are shown. Upper-right panel provides a detailed view of Inner Tracking System (ITS) detectors.

Tracking System (ITS), with six layers of high-resolution silicon detectors, the cylindrical Time Projection Chamber (TPC), a Transition Radiation Detector (TRD) for electron identification, a barrel Time of Flight (TOF), a small-area ring imaging Cherenkov detector at large distance for the identification of high-momentum particles (High Momentum Particle Identifier - HMPID), and two single-arm electromagnetic calorimeters of (Photon Spectrometer - PHOS and EMCal). All central barrel detectors, excluding PHOS, EMCal and HMPID, cover the full azimuthal angle.

The applied field strength is a compromise between momentum resolution and low momentum acceptance. Momentum cut-off should be as low as possible (down to $\simeq 100$ MeV/c), in order to detect the decay products of low-$p_T$ hyperons. At high $p_T$ though, the magnetic field determines the effective momentum resolution, which is essential for the study of high-$p_T$ leptons. Since
the high-pt observables are limited by statistics, ALICE runs mostly with the 0.5 T field option, that is with the maximum field the magnet can produce. The beam pipe is built in beryllium, usually chosen for its lower atomic number and low radiation length $X_0$. It has an outer radius of 3 cm and a thickness of 0.8 mm, corresponding to 0.3% of $X_0$. In terms of radiation lengths, it has the smallest possible thickness to minimize the multiple scattering undergone by the particles produced in the collision.

The muon spectrometer is constructed of an absorber very close to the vertex followed by a spectrometer with a dipole magnet and, finally, of an iron wall to select the muons. It is designed to measure the production of the complete spectrum of heavy quark resonances, namely $J/\psi$ and $\psi'$, $\Upsilon$, $\Upsilon'$ and $\Upsilon''$.

In the forward rapidity region, the set-up is completed by a forward photon counting detector (Photon Multiplicity Detector - PMD) and a multiplicity detector (Forward Multiplicity Detector - FMD) that, in conjunction with the ITS allows the measurement of the charged multiplicity in the range $-3.4 < \eta < 5.1$. A system of scintillators (V0 detector) and quartz counters (T0 detector) provide moreover fast trigger signals.

The collision centrality can be determined by measuring the energy, and thus the number, of spectator nucleons by means of two distinct calorimeters (Zero Degrees Calorimeters - ZDC), made respectively of tantalum and brass with embedded quartz fibers, located on both sides of the interaction region $\simeq 90$ m downstream in the machine tunnel.

In the following, insights will therefore be given to the description of the two most relevant sub-system of ALICE used for the realization of the data sample taken in analysis for this thesis work, namely the ITS and TPC detectors, employed for tracking and reconstruction of the interaction vertex and secondary vertices.

### 3.2 Inner Tracking System

A general view of the ITS is shown in Figure 3.2. The task of the inner tracker is to provide:

- primary and secondary vertex reconstruction with the high resolution
that is required for the detection of hyperons and particles with open charm and open beauty;

- tracking and identification of low-pt particles which are strongly bent by the magnetic field and do not reach the TPC;
- improved momentum resolution for the higher-pt particles which also traverse the TPC.

These goals are achieved with a silicon detector structured in six cylindrical layers, from the inside to the outside: two layers of Silicon Pixel Detectors (SPD), located at \( r = 4 \) and 7 cm; two layers of Silicon Drift Detectors (SDD), \( r = 14 \) and 24 cm; two layers of Silicon Strip Detectors (SSD), \( r = 39 \) and 44 cm.

The first two layers, made up of pixel detectors with a cell (pixel) size of 50 \((r\phi) \times 425 \) \( z \) \( \mu \)m\(^2\), allow excellent position resolution in an environment where the track density may exceed 50 tracks/cm\(^2\). For the two intermediate layers, silicon drift detectors have been selected, since they couple a very good multi-track capability to the information on the specific energy loss with their two-dimensional analog readout. At larger radii, on the two outer layers, the requirements in terms of granularity are less stringent, therefore double-sided silicon micro-strip detectors are used. Double-sided micro-strips have been

\[\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure3-2.png}
\caption{Schematisation of Inner Tracking System.}
\end{figure}\]
selected rather than single-sided ones because they offer the possibility to correlate the pulse height read out from the two sides, a very important aspect for the connection of tracks from the TPC to the ITS.

With the drift and the strip detectors, the four outer layers have analog read-out and may be used for particle identification by applying the method of truncated mean (requiring at least four measurements) for the estimate of the $\frac{dE}{dx}$ of particles below the minimum ionization region, in the $1/\beta^2$ region of the Bethe-Bloch curve. This characteristic makes ITS capable of identifying particles without requiring the contribution of other detectors, that is in a “stand-alone” manner.

One of the main purposes of ITS, is to provide a good precision for the measurement of tracks impact parameter, which is determined by evaluating the so-called distance of closest approach (DCA) to primary vertex. Specifically, the impact parameter projection in the transverse (bending) plane, $d_0(r\phi)$, is defined as:

$$d_0(r\phi) = q \cdot \left( R - \sqrt{(x_v - x_0)^2 + (y_v - y_0)^2} \right),$$

(3.1)

where $q$ is the sign of the particle charge, $R$ and $(x_0, y_0)$ are the radius and the centre of the track projection in the transverse plane (which is a circle) and $(x_v, y_v)$ is the position of the primary vertex in the transverse plane.

For the case of the analysis reported in this thesis work and, more generally, for the measurement of open charm and beauty particles in hadronic decay channels, a good resolution on impact parameter is essential to undertake the separation between primary vertex and secondary decay vertices. Average decay lengths of such particles vary in facts from 100 $\mu$m (as for $D^0$ meson) up to hundreds of $\mu$m for beauty hadrons.

The resolution on impact parameters depends on the resolution of both the prolonged track near the primary vertex and of primary vertex position. Figure 3.3 reports the resolution on impact parameter in the transverse plane, $d_0(r\phi)$, as function of particle transverse momentum, for different kind of particles. It can be noticed how $d_0(r\phi)$ resolution is about $\simeq 75 \mu$m at $p_T = 1$ GeV/c and how particle type is almost non influential for $p_T > 1$ GeV/c. Such a low value is essentially reached because of the noticeable SPD performances.

Since track momentum and position resolutions for particles with small transverse momenta are dominated by multiple scattering effects, the minimiza-
tion of the material thickness is an absolute priority in the ITS, which is the first detector crossed by the particles produced in the collision. Including the carbon-fiber supports and the cooling system, the average material per layer traversed by a straight track perpendicular to the beam line corresponds to 1.2% of $X_0$. Also the drift and strip layers have a similar material budget, so that the total thickness of the ITS corresponds to $\approx 6\%$ of $X_0$.

### 3.3 Time Projection Chamber

The TPC is the main tracking detector in ALICE: it is optimized to provide, together with the other central detectors, track finding, momentum measurement and particle identification via $\frac{dE}{dx}$. A view of the detector is shown in Figure 3.4.

In terms of azimuthal angle, the TPC covers the whole range of $2\pi$ whereas, in terms of measured transverse momentum, it can cover a range from 150 MeV/c up to some hundreds of GeV/c with good effective resolution. It has a cylindrical shape with an inner radius of 80 cm, given by the maximum acceptable hit density ($0.1 \text{ cm}^{-2}$), and an outer radius of 250 cm, given by the length required for a $\frac{dE}{dx}$ resolution better than 10%, necessary for particle identification. The total active length of 500 cm allows the acceptance in the
Figure 3.4 – Shematical view of ALICE Time Projection Chamber.

pseudorapidity range $\eta < 0.9$.

The sensible volume is filled with a gas mixture Ne/CO$_2$/N$_2$ (90%/10%/5%), in which primary ionization electrons drift across a maximum distance of 2.5 m from both sides of the central electrode towards the end-caps. The gas mixture is optimized for drift velocity, low electron diffusion and low radiation length, whereas the drift field is set to about 400 V/m, providing a maximum drift time of 90 $\mu$s. The TPC readout chambers are multi-wire proportional chambers with cathode-pad readout. The readout planes at the two ends of the large drift volume ($\approx 88 m^3$) are azimuthally segmented in 18 sectors, each covering an angle of 20deg. The radial thickness of the detector is of 3.5% of $X_0$ at central rapidity and grows to $\approx 40\%$ towards the acceptance edges.

For this thesis work purposes, TPC will be used as main detector for electrons identification. The analog read-out of TPC pads allows in facts particle identification through measurements of $dE/dx$, either in the low momentum region or in the relativistic rise region of Bethe-Bloch curve. Figure 3.5 is referred to an actual example of TPC particle identification in $p - p$ collisions at $\sqrt{s} = 7$ TeV. It reports the measured specific energy loss in the TPC active material as function of transverse momentum along with the parametrized Bethe-Bloch curves for the most relevant particles. As it can be noticed, TPC allows a good pion/electron separation up to momenta of $\approx 7$ GeV/c.

Most of limitations for TPC are due to the fact that at high interaction rates
the spatial charge produced by the overall tracks ionization in the TPC volume may generate an electric field which is comparable with that used for drifting, thus possibly distorting reconstructed tracks of up to some millimeters. In order to avoid this problem, it is needed to account of a TPC “memory” time, which is essentially due to ionized charge drift time (up to \( \approx 90 \, \mu s \)), and consequently limit beam luminosity to reduce interaction rate.
Chapter 4

Extraction of non-prompt $J/\psi$

Already before the start-up of LHC, experiments carried out at the Tevatron CDF on inclusive $J/\psi$ production in $p\bar{p}$ collisions up to $\sqrt{s} = 1.96$ TeV revealed how a significant fraction of the yield was made up of $J/\psi$ resulting from the decay of beauty-flavoured hadrons [51]. A similar picture in a higher energy domain was naturally expected for LHC, whose experiments extensively studied $J/\psi$ production in $p-p$ collisions at $\sqrt{s} = 7$ TeV in the following years. In order to distinguish it from the so-called “prompt” component of the yield, made up of all those $J/\psi$ directly produced in the primary interactions or resulting from prompt decays of higher excited charmonium states (such as $\xi_c$ and $\psi(2S)$), this component is often labelled as “non-prompt” and is characterized by being produced at a relatively large distance from the primary interaction vertex.

As discussed in section 2.4, extracting the non-prompt component actually arises as an important aim of inclusive $J/\psi$ analyses, not only because it allows the measurement of the $J/\psi$ prompt-production cross section, relevant for the testing of several QCD predictions, but also because it is directly linked to beauty-flavoured hadrons production, which are main object of study of heavy-flavour physics and have proven to be valuable tools for investigating heavy-ion collisions and related phenomena. Purpose of the work, has been to extract the non-prompt fraction $f_B$ from a data sample of inclusive $J/\psi$ candidates reconstructed from their electromagnetic decay channel $J/\psi \rightarrow e^+e^-$ in a collection of minimum-bias $p-Pb$ collisions at the c.m.s. energy per
Extraction of non-prompt \(J/\psi\) nucleon pair of \(\sqrt{s_{NN}} = 5.02\) TeV in the central rapidity region \(|y_{J/\psi}| < 0.9\) of the ALICE experiment at LHC. The obtained results should be considered as raw preliminary values which may though prove to be a valuable basis for the further and more detailed incoming studies.

The whole work essentially relied on the baseline provided by the previously performed ALICE analysis on inclusive \(J/\psi\) production in \(p - p\) collisions reported in [52]. It employed indeed the same adopted separation technique, based on a statistical separation of the yield components by means of an un-binned maximum likelihood fit which has been properly adapted from the latest official ALICE analysis code repository.

Throughout the chapter a detailed step-by-step explanation of the the above-stated method will be furnished. The starting point will be the description of the examined data sample, with insights on its acquisition and the exposition of the performed experimental cuts. In the second section I will introduce the principles as well as define the relevant observables on which the employed separation technique is founded, whereas the third section will be dedicated to the full exposition of the likelihood fit and of its specific application to the performed analysis. Lastly, in the fourth section I will report and comment the various fits performed for the estimation of the necessary parameters which were used for the statistical extraction of the non-prompt fraction.

All the algorithms, codes, fits and computations employed throughout the work were developed and performed by means of ALICE’s AliRoot analysis and simulation environment, based on the object-oriented ROOT analysis framework. The final result, along with the procedures employed for the estimation of its statistical uncertainties, will be reported in the next and final chapter.

\section{Analysed Data Sample}

The data sample taken in analysis for the work was in the form of a ROOT Tree data file, consisting of a collection of kinematic and other relevant physical values computed on a set of candidate \(J/\psi \rightarrow e^+e^-\) decay tracks within ALICE central barrel. These tracks were reconstructed on the basis of a much larger set of “Minimum-Bias” \(p - Pb\) collision events collected over a data taking period which covered the first two months of the year 2013. The analysed
Extraction of non-prompt $J/\psi$

Sample is indeed the ultimate product of a long and well-established selection procedure, aimed at providing high quality measurements out of the whole set of “raw” data coming out of the experiment detectors signals. A long step-by-step selection is made, in facts, on the physical event data collected by the detectors in order to filter out a subset of data optimized for analysis purposes. Furthermore, a series of quality cuts is then applied to the reconstructed tracks, in order to return data samples that can be considered as well-suited for the specific analysis. In this section, I will provide insights on both of these kind of cuts, for the specific case of the analysed data sample.

4.1.1 Event selection

The rawest set of analysable data is called Event-Summary-Data (ESD). It comes after a procedure of global event reconstruction, which converts the signals coming from detectors storing all the event relevant information about tracks, particles and global features. This raw sample is subject to a step-by-step procedure which gradually improves its content in terms of quality and physical significance.

In order to extract the sample of “Minimum-Bias” events for the performed analysis, a trigger-selection was applied to ESD by requiring that each event verified the condition of having a non-zero signal in both VZERO detectors. These are namely the two scintillators V0A and V0C, located along the beam directions at $z = 3.4m$ and $z = -0.9m$ and covering the pseudo-rapidity regions $2.8 < \eta < 5.1$ and $-3.7 < \eta < -1.7$ respectively. In a physical collision event, these are supposed to be hit by particles resulting from a fragmentation of one of the two colliding projectiles. The applied coincidence trigger is then meant to increase the probability of selecting an inelastic collision between the proton and the lead nucleus, which is of course the one of physical interest. The data class of minimum-bias events resulting after such a selection is called KINT7 and, for the specific case, counted about 122.46 millions of entries. A number of further cuts, which is usually referred as “physical selection”, is then applied to the sample in order to optimize its content in terms of event topology and reliability. Among these is, for example, the requirement, for
Extraction of non-prompt $J/\psi$ events sharing a similar topology, of having a good correlation between the number of tracks and number of hits in the SPD layers of ITS. The resulting sample after the whole physical selection procedure was made up of about 121.67 millions of events.

For the specific case, a further event cut was also applied at this step by requiring the event reconstructed primary vertex position component along the beam direction $z_V$ to satisfy the condition $|z_V| < 10 \text{ cm}$.

4.1.2 Track selection

Before proceeding with the specific selections for extracting the $J/\psi$ candidates, the selected events were furtherly filtered out by requiring them to contain good quality tracks. The series of standard selection criteria applied at this step typically makes use of parameters estimated during the track reconstruction procedure in ITS and TPC.

In the specific, tracks were required to have a minimum number of associated TPC clusters $N_{cls} > 70$ and a $\chi^2$ per space point for the momentum fit lower than 4. Furthermore, they were required to point back at the interaction vertex within a specific maximum distance of closest approach (DOA), namely within 3 cm along the beam direction and within 1 cm in the transverse plane. A successful global refit, from the outer layer to the primary vertex, incorporating TPC and ITS space points was also required, and tracks qualified as originating from “kink” decays were rejected.

A data sample resulting from the application of the above-mentioned general selection criteria is called Analysis-Object-Data (AOD). It contains much higher quality events with respect to the original ESD sample, and proves to be suited for most analyses at central rapidities.

4.1.3 $J/\psi$ candidates selection

Further cuts applied on AODs are mostly driven by analysis purposes. For the specific case, $J/\psi$ candidates were selected out of the filtered AOD sample by applying kinematical and particle identification criteria.

Firstly, kinematic cuts were applied. Candidate $e^+e^-$ tracks were required to
Extraction of non-prompt $J/\psi$

do not hallucinate.

possess a minimum transverse momentum of $p_T^{e^+e^-} > 1$ GeV/c, and were furthermore rejected if outside of the detectors central acceptance region $|\eta^{e^+e^-}| < 0.9$. This selection is meant to provide a better momentum resolution for the reconstructed $J/\psi$ candidates.

Particle identification (PID) of electrons was performed via specific energy loss $\frac{dE}{dx}$ measurements in TPC. Tracks were considered as electron candidates if their specific energy loss as function of momentum was compatible with the value of Bethe-Bloch fit for electrons. An inclusion cut in terms of statistical deviations was then applied by requiring the converted TPC signal to lie within $\pm 3\sigma$ around the theoretical value. Most of contaminations arising from the application of this method are due to the contributions of the close pions and protons Bethe-Bloch distributions in the mid-momentum region, therefore two further exclusion cuts were applied. In the specific, a $\pm 3\sigma$ exclusion cut was applied for both proton and pions fit value.

For the above identified particles, a combinatorial algorithm properly combines opposite charge tracks in order to reconstruct a physical $J/\psi$ candidate. Most of the combinatorial background at this step arises from conversions of photons into an electron pair $\gamma \rightarrow e^+e^-$. A number of further preliminary cuts is then applied to reduce this kind of contribution. Since most of the conversions are expected to occur when photons cross the material layers of the detectors, a first selection is applied by requiring all the tracks to have one hit in the first layer of SPD detector. This selection actually results also in a significant improvement of the spatial resolution for the measurement of $J/\psi$ decay vertex position. A “V0” algorithm then rejects the tracks topologically compatible of being daughters of neutral particles and whose decay vertex lie within the most probable regions for $\gamma$ conversion. A further selection is made among the combined opposite charge tracks by rejecting the pairs whose invariant mass resulted compatible with 0 (more precisely, an invariant mass cut of $|M_{e^+e^-}| < 100$ MeV/c$^2$ was applied. Finally, among the resulting pairs, one last kinematic cut is employed by requiring the mother’s $J/\psi$ rapidity, as measured in the laboratory frame, to lie in the central region $|y^{J/\psi}| < 0.9$.

Out of the 107.12 millions of event passing the full event selection procedure, a total number of 33201 $J/\psi$ candidates was present in the final data sample.
Figure 4.1 – Distributions of the number of dielectron pairs with respect to their absolute distance, expressed in number of statistical deviations $\sigma$, from the theoretical values of pions (up-left, up-right) and protons (down-left, down-right) Bethe-Bloch fits. Histograms are filled considering only opposite charge pairs.

The candidates sample, as anticipated, was analysed in the form of a ROOT Tree data file, which stored, for each candidate, all the relevant physical informations on both the daughter electron-positron tracks, usually referred as legs, and on the reconstructed mother $J/\psi$. The two tracks for each dielectron pair were moreover categorized as leg1 or leg2 according to which of them possessed the greater momentum.

In the following figures, as a quantitative example, I will report some relevant distributions which were directly computed from the analysed data sample.

At first, I report in Figure 4.1 the distributions of the number of dielectron legs with respect to their absolute distance, expressed in number of statistical deviations $\sigma$, from the theoretical values of protons and pions Bethe-Bloch fits. As can be easily seen, all distributions show the above-mentioned exclusion cut at 3.0 $\sigma$ employed for PID. Differences in shapes between the two legs distributions are due to the different momentum content of each leg, and hence to
Extraction of non-prompt $J/\psi$

the different average position in the relative Bethe-Bloch fits.

Figure 4.2 - Bi-dimensional distributions of the number of dielectron legs with respect to both their pseudorapidity $\eta$ and their statistical distance, expressed in terms of standard deviations $\sigma$, from the Bethe-Bloch fit for electron. All pairs are enclosed in the $[-0.9,0.9] \times [-3.0,3.0]$ rectangle of the $\eta-\sigma$ plane as a consequence of the applied cuts. Black markers indicate the mean values of the sigma distributions calculated in each pseudorapidity bin. Only opposite charge pairs are reported.

Figure 4.2 shows the legs distributions with respect to both their pseudorapidity $\eta$ and their statistical distance, again in terms of standard deviations $\sigma$, from the Bethe-Bloch fit for electron. Now both the discussed kinematic cut $|\eta^{e^{+},e^{-}}| < 0.9$ and PID $\pm 3\sigma$ inclusion cut are clearly visible. Black markers indicate the mean values of the sigma distributions calculated in each pseudorapidity bin. Their almost constant trend around 0 as function of rapidity is an indication of the good quality of the performed selections.

Lastly, Figure 4.3 reports the opposite charge dielectron pairs invariant mass $M^{e^{+}e^{-}}$ distribution, plotted within the invariant mass region between 1.0 and 6.0 Gev/$c^2$, without any further cuts applied. The peak at $M^{e^{+}e^{-}} \simeq 3.1$Gev/$c^2$, corresponding to the $J/\psi$ resonance, arises clearly visible above the combinatorial background.
I shall stress that all the histograms reported until now, are filled considering only the reconstructed “opposite-sign” (OS) pairs, that is only those pairs whose legs have opposite charge and are hence physically eligible of being product of a $J/\psi$ decay. The analysed data sample provided though also a certain number of “like-sign” (LS) dielectron pairs, namely combined $e^+e^+$ and $e^-e^-$ pairs, which are not considered as physically relevant $J/\psi$ candidates, but which prove to be particularly useful, as it will be seen in section 4.5, for the estimation of the combinatorial background shape. If not expressly specified, all the plots reported from now on will account only for OS pair contributions.

### 4.2 Separation of prompt and non-prompt $J/\psi$

At hadronic colliders, the totality of measured $J/\psi$ resonances may be considered as originating from three main different processes:

- **direct production**: when the $c\bar{c}$ pair produced in a hard scattering process...
directly combines to form a $J/\psi$ state.

- **decay from higher excited charmonium states**: when the produced $c\bar{c}$ pair hadronizes in a higher mass state than $J/\psi$, such as a $\chi_c$ or a $\psi(2S)$ state, which subsequently decays into a $J/\psi$ state.

- **decay from beauty-flavoured hadrons**: when the observed $J/\psi$ is a product of a weak decay from beauty flavoured hadrons $H_B$, coming after the hadronization of a $b\bar{b}$ pair originally produced in the initial hard scattering processes.

$J/\psi$ resulting from the first two types of processes are defined as *prompt* $J/\psi$, since their production occurs in both cases “right after” the collision, i.e. at a distance which is experimentally not distinguishable from the primary interaction vertex position. On the other hand, $J/\psi$ resulting from the last process, are product of a weak decay ($b \rightarrow cW$) process, which generally takes a much longer time to occur, and are hence produced at a measurable distance from the primary vertex position. More precisely, the *proper decay time* $\tau$ of a beauty-flavoured hadron is of the order of some picoseconds, which means that the hadron will travel a distance $\sim c\tau$ in the laboratory which is of the order of hundreds of $\mu m$ before decaying into a $J/\psi$ state.

All of the above produced $J/\psi$ may then decay into a an $e^+e^-$ pair, through the annihilation of the constituent $c$ and $\bar{c}$ quarks. Since $J/\psi$ meson is characterized by a decay width of 92.9 KeV, electron tracks won’t be experimentally separable from the $J/\psi$ production site. As a consequence, reconstructed di-electron tracks resulting from the decay of prompt $J/\psi$ states will be seen as coming directly out of the primary vertex, whereas the ones resulting from non-prompt $J/\psi$ decays will point back to a secondary vertex which falls at an experimentally resolvable distance $L$ from the primary collision. Exploiting such a “physical” separation is actually the most natural way to carry out the sought “statistical” separation of the two components of the yield.

One must though notice that the experimentally measured separation $L$ alone is simply not enough to achieve a quantitative separation which correctly takes into account the fact that neither the masses nor the momenta of the decaying b-hadrons are known, as well as the fact that all the physically relevant
information about the observed $J/\psi$ are inferred from the reconstructed electron pairs resulting from their decays. These two different lacks of knowledge are actually a natural consequence of the experimental analysis which is being performed. The first effect is due to the fact that the analysis is “inclusive” in nature, in the sense that it does not care about identifying which kind of beauty-hadron has decayed in the observed $J/\psi$, but only about estimating how many of these $J/\psi$ are product of such kind of decays. Each of these unknown hadrons will randomly decay in the laboratory at a different time, according to both its mass and to the relative time dilatation induced by its momentum, consequently spreading the measured decay length $L$ distribution.

The second fact arises as a direct consequence of the natural limitations of the experiment. Even considering the un-physical case in which all of the candidate electron pairs are the actual product of a $J/\psi$ decay, i.e. that no “background” is present in the experiment, the experimental uncertainties affecting their reconstructed tracks and momenta will impair the measurement of the $J/\psi$ kinematic variables, as well as the capability of identifying a secondary decay vertex if not within a certain resolution distance. As discussed in chapter 3, this is particularly relevant for both high-momentum particles, whose resolution is impaired by the small curvatures of their tracks, as well as for very low-momentum particles, affected by multiple scattering effects when crossing the material layers of the detectors. For what concerns the performed analysis, the latter effect happens to be predominant, essentially because the applied PID techniques, based on specific energy loss measurements in TPC, do not allow high momentum electrons ($Pt > 10$ Gev/c) to be correctly identified and subsequently collected.

A precise statistical technique, relying on well-defined observables, which can account properly for the experimental resolution uncertainties as well as of effective knowledge on the measured objects, efficient in rejecting any background contribution, must therefore be defined for the purpose.

What I am to do in this section, is to define and comment the observables on which such a separation technique relies on. These are actually the same observables measured in the previously performed analyses on non-prompt $J/\psi$ yield in $p - p$ experiments, and will serve as variables for the employed sta-
Figure 4.4 – Graphical example for the calculation of the $L_{xy}$ variable. $x - y$ is the transverse plane of ALICE global reference frame and $B$ stands for the production site of the $b$-hadron. $L_{//P_{T}^{B}}$ is the transverse projection of the flight distance $L$ of the $b$-hadron, whereas the $\theta$ angle indicates the transverse projection of the opening angle between the $J/\psi$ flight direction and that of the $b$-hadron.

The starting point is to introduce the scalar variable $L_{xy}$, defined as the signed projection of the $J/\psi$ flight distance onto its transverse momentum vector, $\vec{p}_{T}^{J/\psi}$:

$$L_{xy} = \vec{L} \cdot \frac{\vec{p}_{T}^{J/\psi}}{|\vec{p}_{T}^{J/\psi}|}.$$  \hspace{1cm} (4.1)

Here $\vec{L}$ denotes the measured vector pointing from the primary vertex to the $J/\psi$ decay vertex and is assumed to be best estimate of the $b$-hadron travelled decay length. Figure 4.4 provides a concrete example for the evaluation of $L_{xy}$. One may immediately notice how the projection is “signed”, in the sense it that can assume either positive or negative values, according to the separation between the $b$-hadron flight direction and $J/\psi$ momentum component in the transverse plane. Being $J/\psi$’s the heaviest product of $b$-hadrons decays, they are expected to carry the largest fraction of $b$-hadrons longitudinal momentum, typically travelling nearly collinear with respect to their flight di-
Extraction of non-prompt $J/\psi$ correction. A strong correlation is hence expected between non-prompt $J/\psi$'s and high-transverse momentum b-hadrons, resulting in large and positive measured values of $L_{xy}$ in the transverse plane. A non-negligible amount of $J/\psi$ with large opening angle between their flight direction and that of the b-hadron is though present, and this consequently results in the spreading of $L_{xy}$ towards negative values for lower transverse momentum $J/\psi$. Such a strongly $p_T^{J/\psi}$ dependent variable is then still not suited for the separation purpose, but may actually be used to define an observable of much greater physical relevance. One defines the so called pseudo-proper decay length $x$ as:

$$x = L_{xy} \cdot \frac{M_{J/\psi}}{p_T^{J/\psi}},$$

(4.2)

where $M_{J/\psi}$ is the world average $J/\psi$ mass. The name arises from its strict connection to the proper decay time $\tau$ of the b-hadron. It is easy to notice, in facts, that if the actual mass $M_{H_b}$ and momentum $p_{H_b}$ values of the decaying b-hadron $H_b$ were considered in place of the respective $J/\psi$ values in both equations (4.1) and (4.2), then the $x$ variable would be equal to the b-hadron proper decay length:

$$c\tau = \frac{L}{\beta\gamma} = L_{xy} \cdot \frac{M_{H_b}}{p_T^{H_b}}.$$  

(4.3)

The so defined variable hence happens to be of great use since it actually “mimics” the average proper decay length of the decaying b-hadrons, expected to be of the order of hundreds of microns. The term “pseudo” is to indicate that $J/\psi$ parameters are used as a substitute of the unknown $H_b$ values in its definition. This is actually a consequence of the fact that in the present analysis b-hadrons are not reconstructed exclusively and that therefore their masses and momenta can’t be measured. Despite of such a “hybrid” definition, the $x$ variable is actually a good guess of the actual $c\tau$ value and will be used as key-observable in the employed separation method. Prompt and non-prompt $J/\psi$ behave effectively in a very different way with respect to such a variable. Prompt $J/\psi$ are produced in the proximity of the primary interaction vertex, therefore their decay length is $L \approx 0$ and their resulting $x$ distribution should approximate a Dirac delta centred at $x = 0$. On the other hand, non-prompt $J/\psi$ are the result of b-hadrons decays, which are characterized by measurable non-zero decay lengths. One may then expect, in
virtue of the above-discussed features, that their $x$ distribution should appear shifted towards positive values, quantitatively expressing the b-hadron mean travelled length before its decay into a $J/\psi$.

Figure 4.5 provides a convenient example of such two very different trends, extracted from a Monte Carlo simulation in which both prompt and non-prompt $J/\psi$ decay events were reconstructed in a number of simulated $p-Pb$ collision at ALICE detector (more details will be given about the analysed Monte Carlo simulated data in section 4.4). The simulated $J/\psi$ measurements are plotted as function of the pseudo-proper decay length in the range $-3000 < x < 3000 \mu m$, after the application of a minimum transverse momentum cut at $p_T^{J/\psi} > 1.0$ GeV/c. Distributions were furthermore normalized to unity in order improve the qualitative comparison of their shapes.

A number of remarkable features common to experimental observations, may be pointed out by looking at simulated data. The clear aspect is of course that $x$ variables exhibits two very different behaviours for prompt and non-prompt
Extraction of non-prompt $J/\psi$. As one could imagine, prompt component doesn’t really distribute according to a Dirac delta because of experimental uncertainties affecting the identification of dielectrons decay vertex and the consequent reconstruction of both $L_{xy}$ and $x$. The result can be explained as if affected by an effective resolution factor, which symmetrically spreads the distribution towards larger $x$ values. A very different trend is though exhibited by the non-prompt $J/\psi$ component, whose $x$ distribution, appears as completely shifted towards positive $x$ values.

Such an asymmetry will be the key-feature to be exploited in order to carry out the yield separation, but a number of restraining aspects must preliminarily be considered. Firstly, one can notice how the detectors experimental resolution plays a decisive role in the separation efficiency. Both components are in principle affected by resolution effects, and the more such effects get significant, as for the case of very low momentum ($P_t < 1$ GeV/c) reconstructed tracks, the more prompt and non-prompt distributions will get similar and hence less distinguishable. Achieving a good experimental resolution is therefore an actual requirement for the analysis, whose implications will be discussed in detail in the following sections.

Non-prompt component gets also impaired when low-momentum $J/\psi$ are considered. This is a consequence of the mentioned non-negligible fraction of $J/\psi$ with large opening angle between their flight direction and that of the b-hadron, whose effect manifests in figure as an exponential tail falling over negative $x$ values. Figure 4.6 can provide a qualitative estimate such kinematic contribution. Non-prompt $J/\psi$ pseudo-proper decay length distributions are extracted from the same Monte Carlo simulated data and plotted within the same $x$ range for different values of minimum $p_{TJ/\psi}$ cut. The entity of the kinematic effect is evident for $J/\psi$ with momentum lower than 1 GeV/c, resulting in much more symmetric distributions. At about $p_{TJ/\psi} > 2$ GeV/c, effect is minimal, and the residual slopes at negative $x$ can be considered as due to resolution effects only.

All the above-mentioned features must be taken into account before any use of $x$ as statistical separation variable is made. A necessity arises then of a minimum transverse momentum cut in order to improve the variable separation power. Such a cut was chosen, as will be explained, at the same value
Extraction of non-prompt $J/\psi$

employed in previous ALICE $p - p$ analyses, namely at $p_T^{J/\psi} > 1.3 \text{ GeV}$.

To get a more quantitative insight on the performed task, I finally report the actual pseudo-proper decay length distributions extracted from the analysed data sample. Distributions are plotted in Figure 4.7 for different applied minimum $p_T^{J/\psi}$ cuts, and all within the same invariant mass window $2.4 < M_{e^+e^-} < 4.0 \text{ GeV/c}^2$ around the signal peak. This time histograms show the overall experimental distributions which include naturally both kinds of $J/\psi$ but also, and most important, combinatorial background. If no $p_T$ cuts are applied (upper-left frame), distribution is almost completely symmetric, and a separation of the components seems nearly impossible at first sight. Higher $p_T^{J/\psi}$ cuts show, on the contrary, a slowly increasing asymmetry which is due to the increasing fraction of $J/\psi$ from beauty-hadrons. For $p_T^{J/\psi} > 5 \text{ GeV/c}$, despite the low events statistics, the non-prompt component appear significant. The momentum correlation between $J/\psi$ and $b$-hadrons in the

![Figure 4.6 - Non-prompt $J/\psi$ pseudo-proper decay length distributions, extracted from Monte Carlo simulations, plotted for different values of applied minimum $p_T^{J/\psi}$ cut. For events with very low $p_T^{J/\psi}$, the non-negligible amount of $J/\psi$ with large opening angle between its flight direction and that of the $b$-hadron impairs the separation ability, symmetrizing the distributions. Effect becomes negligible at about $p_T^{J/\psi} > 2 \text{ GeV/c}$.]
transverse plane increases with increasing $p_T^{J/\psi}$, hence the hard $p_T^{J/\psi}$ yield component will essentially reproduce the hard momentum spectrum of the heavier b-hadrons. The above-discussed kinematic and resolution effects are also less important at high $p_T$, resulting in narrower and more pronounced asymmetric distributions.

Background component is particularly impairing for analysis. Its presence can be especially noticed at large $x$ values, where it manifests as a rather uniform distribution of spurious entries. The effective signal-over-background ratio $S/B$ depends though on the invariant mass range taken in consideration. Taking as reference Figure 4.3, one may realize how focusing analysis on narrower mass windows around the signal peak effectively enhances the $S/B$ ratio, but
limits the amount of information which can be inferred on background $x$ distributions. On the other hand, analyses on wider mass ranges provide better statistics and more background information, but shadow signal presence and impair its consequent separation. Identifying and estimating the statistical contribution from background constitutes the most difficult task to perform for the correct extraction of $J/\psi$ components.

4.3 Un-binned Maximum Likelihood Fit

Pseudo-proper decay length has been shown to be an information-rich observable, capable of distinguishing between prompt and non-prompt components of an inclusive $J/\psi$ yield. The purpose of this section is to explain now how such observable can actually be employed in a well-defined procedure in order to achieve a statistical separation of these components from an experimental data sample.

The employed procedure was originally introduced by CDF [51] and is actually still used by all other LHC experiments\(^1\) in inclusive $J\psi$ analyses. It essentially makes use of an un-binned bidimensional fit, performed on both the invariant mass $m_{e^+e^-}$ and pseudo-proper time $x$ distributions, applying the statistical method of maximum likelihood estimation to extract the fractions of signal and non-prompt component from a data sample, within a specified invariant mass interval.

Before explicating the method itself, it is convenient to recall the statistical principles on which the maximum likelihood estimate is founded. Essentially, maximum likelihood is a mathematical inference procedure which corresponds to one of the so-called estimation methods in statistics, used to provide estimates for the unknown statistical parameters characterizing the underlying statistical distribution of a data set. In principle, for a given data sample of $N$ independent observations, supposed to be distributed according to an underlying statistical model, the method of maximum-likelihood maxi-

\(^1\)Only LHCb, which detects particles only at forward rapidities, employs a different procedure, making use of the longitudinal momentum dependent variable $t_z = \frac{M_{J/\psi}}{p_z}$ in place of $x$. 
mizes the “agreement” of the selected model with the observed data. In other words, if the underlying model provides statistical predictions on the probability to observe a fixed set of data but the model parameters are unknown, the likelihood method selects the “best” set of values of the model parameters by maximizing the probability of observing the given data under the resulting distribution.

The so-called joint density function describes the probability of observing a whole given set \((x_i)_{i=1,...,N}\) of \(N\) observations of a random continuous variable \(x\). If one supposes such observations to be independent and identically distributed according to an unknown one-parameter probability density function \(f(\cdot|\theta)\), then can express the joint density function as the product

\[
f(x_1, x_2, \ldots, x_N | \theta) = f(x_1|\theta) \cdot f(x_2|\theta) \cdots f(x_N|\theta).
\]

If now the “true” parameter \(\theta_0\) of \(f\) is unknown and only the observed sample is provided, the function can be looked from a different perspective by considering the observed values \(x_i\) to be its fixed “parameters” while \(\theta\) as its free variable. The resulting function:

\[
\mathcal{L}(\theta | x_1, \ldots, x_N) = f(x_1, x_2, \ldots, x_N | \theta) = \prod_{i=1}^{N} f(x_i|\theta).
\]

is called the likelihood function.

The method of maximum likelihood provides the best estimator \(\hat{\theta}\) of the true parameter \(\theta_0\) by selecting a value of \(\theta\) that maximizes the likelihood function. In practice, being logarithm a monotonically increasing function, it is often more convenient to work with the logarithm of the likelihood function

\[
\ln \mathcal{L}(\theta | x_1, \ldots, x_n) = \sum_{i=1}^{N} \ln f(x_i|\theta),
\]

since the resulting best estimator will be the same, if any exists.

The former principles can now be directly applied to the analysed data sample, in the specific, to the set of \(N\) independent observations of mass and pseudo-proper decay time \((x_i, m_i^{+}e^{-})_{i=1,\ldots,N}\) falling within a given invariant
Extraction of non-prompt $J/\psi$ mass interval. The probability of measuring a couple of values $(x, m_{e^+e^-})$ will depend on the nature of the observed candidate, e.g. on whether the candidate is a prompt or non-prompt $J/\psi$ or even on whether the candidate is a real $J/\psi$ or a background combination. Furthermore, having observed a specific value of one of either $m_{e^+e^-}$ or $x$ for a candidate, does not condition in principle the possible values that the other variable can assume.

The set of experimentally measured couples $(x_i, m_{e^+e^-}^i)_{i=1,...,N}$, may then reasonably be considered as the randomly extracted sample of two independent variables statistically distributed according to an overall underlying probability density function $F(x, m_{e^+e^-})$, which generally accounts for all of the different candidate proprieties. The signal and background fractions within the chosen invariant mass interval, as well as the fractions of prompt and non-prompt $J/\psi$ components, are expected to characterize the probability distribution of both $m_{e^+e^-}$ or $x$, and may therefore be considered as unknown parameters of resulting probability density function. The method of maximum likelihood happens then to be a well defined statistics-driven approach for the estimation of such parameters, given the observed data sample, and provided the separation capability granted by the $x$ and $m_{e^+e^-}$ observable.

The task can then be summarized in maximizing of the log-likelihood function:

$$
\ln \mathcal{L} = \sum_{i=1}^{N} \ln F(x_i, m_{e^+e^-}^i), \quad (4.4)
$$

where $N$, as said, is the total number of measured candidates in the chosen mass window, and the function $F$ is evaluated in each observed value $(x_i, m_{e^+e^-}^i)$. Contrarily to the previous example, in this case $F$ is a bidimensional density function, describing simultaneously the probability of observing both $x$ and $m_{e^+e^-}$, and will in general depend on an array of parameters.

For the above-mentioned considerations, assuming $x$ and $m_{e^+e^-}$ as independent variables, the function $F$ can be written, in very general terms, as a statistical mixture of products of one-dimensional density functions:

$$
F(x, m_{e^+e^-}) = f_{\text{Sig}} \cdot F_{\text{Sig}}(x) \cdot M_{\text{Sig}}(m_{e^+e^-}) + (1 - f_{\text{Sig}})F_{\text{Bkg}}(x) \cdot M_{\text{Bkg}}(m_{e^+e^-}) \quad (4.5)
$$

where the sum accounts for the candidates possibility to be a real $J/\psi$ or a background combination, and that each of these cases is described by different
Extraction of non-prompt $J/\psi$

probability distributions. In the specific:

- parameters $f_{\text{Sig}}$ and $(1 - f_{\text{Sig}}) = f_{\text{Bkg}}$ represent the overall fractions of signal and background candidates in the given mass interval ($f_{\text{Sig}}$ fraction is is coincident with the ratio $S/(S + B)$).

- $F_{\text{Sig}}(x)$ and $F_{\text{Bkg}}(x)$ indicate the probability density functions describing the pseudo-proper decay length $x$ distribution of signal and background candidate $J/\psi$ respectively.

- $M_{\text{Sig}}(m^{e^+e^-})$ and $M_{\text{Bkg}}(m^{e^+e^-})$ are the probability density function describing the respective signal and background candidates invariant mass $m^{e^+e^-}$ distributions.

All of the former one-dimensional density functions, are normalized to unity within the infinite range of their variability. Their integral computed within a finite interval is by definition the probability of randomly extracting a value of their variable within the same interval.

In particular, the signal part of pseudo-proper decay length function, $F_{\text{Sig}}(x)$, must account for the different behaviour of prompt and non-prompt $J/\psi$ with respect to $x$ and can be further factorized as:

$$F_{\text{Sig}}(x) = f_B \cdot F_B(x) + (1 - f_B) \cdot F_{\text{prompt}}(x).$$  \hspace{1cm} (4.6)

Where this time, $F_B(x)$ and $F_{\text{prompt}}(x)$ stand for the non-prompt and prompt probability density function, whereas $f_B$ indicates the overall fraction of $J/\psi$ resulting from b-hadrons decays.

Ideally, the functional shape of each probability density function is coincident with that of the distribution of an infinite number of observations, plotted within infinitesimally-small intervals. The statistical procedure consists then to parametrize each the former functional shapes with a number of reasonable parameters, and subsequently to extract the non-prompt fraction $f_B$ by estimating the values of such parameters via the method of maximum likelihood estimate.

Although maximum likelihood method is in principle capable of selecting the best set of estimators, the actual functional shapes are rather complex and the final parametrizations employed for the purpose accounted a total number
of more than 30 parameters. An simultaneous un-constrained fit of all the parameters would then be a reasonably impracticable task. The developed strategy therefore was slightly different. In order to provide more constraints for the final maximum likelihood fit, the work was subdivided by singularly fitting each of the one-dimensional density functions. The parameters of each function were to be evaluated and consequently fixed by means of binned $\chi^2$ fits on the measured distributions, so that eventually only the $f_B$ fraction, or possibly a few other parameters (such as $f_{Sig}$ or some background weights), could be evaluated with the un-binned likelihood fit. The following sections will report in the detail the functional parametrizations employed for the task, as well as the results of the performed fits on the relative binned distributions and the choices made for selecting the finally examined candidate sample.

4.4 Binned Plots Fit

Maximum likelihood method allows the estimation of the non-prompt component of the measured $J/\psi$ yield on the direct basis of the experimentally measured sample, provided the knowledge of the underlying statistical models. Such models are actually not known *apriori*, but may as well be inferred from observations on the basis of some reasonable assumptions. The principle is to assume an arbitrary parametrization for each of the relevant probability functions and subsequently to check their reliability by means of binned $\chi^2$ fits on the relative experimental distributions. It essential to remark that a good $\chi^2$ fit value should be considered as a signature of a good parametrization choice *only* for the analysed data points. Nonetheless, if data are properly plotted and if the number of measurements is sufficiently high, then the “best fit” curves resulting from $\chi^2$ minimization may ideally be considered, in shape, as a good approximation of the underlying probability density function, whose parameters are to be extracted. How to properly select and plot data, in a way that they could provide as much information as possible about the unknown probability functions, can be considered as the real task of the performed work.
4.4.1 Preliminary cuts

A proper selection of the candidates sample is not only a convenient, but a discriminant choice which has to be performed to ensure the successful outcome of the whole procedure. Firstly, even if maximum likelihood fit makes no use of binned plots, dealing only with probability functions generally defined on infinite domains, it actually requires the preliminary choice of a finite invariant mass working range. In other words, only the couples \((x_i, m_i^{e^+e^-})\) within the specified interval will be considered to produce a statistical estimate on their distribution \(F(x, m^{e^+e^-})\). The reason is that the fraction \(f_{\text{Sig}}\), used to define the relative weights of signal and background components in equation 4.5, only have sense if referred to a specific mass range. From a more practical perspective, however, the choice is also a compelling necessity for a reasonable separation of the two components themselves. Dealing with high sample statistics improves the significance of the final result, so that large operative mass ranges may look as the best choice to make, but a large mass range also implies smaller signal fractions to deal with. A narrow mass window around the signal peak, on the other hand, improves the S/B ratio but reduces sample statistics and may also impair the evaluation of \(x\) background distribution.

The final invariant mass range was chosen for analysis to be between 2.4 and 4.0 GeV/c\(^2\). This choice revealed to be of convenient use, not only because of the good compromise between candidates statistics and signal component, but also because invariant mass background shape within the same range appeared as described by a much simpler parametrization.

A further selection was then considered by evaluating the application of transverse momentum cuts. Even if not expressly present in equation 4.5, both \(f_{\text{Sig}}\) and \(f_B\) are naturally determined by the average transverse momentum of the sampled candidates. The application of a \(p_T\) cut to the examined sample, as anticipated, happens also to be of great relevance for the resulting pseudo-proper decay length distributions, especially for what concerns experimental resolution effects. In particular, the measured prompt \(J/\psi\) \(x\) distribution, \(F_{\text{prompt}}(x)\), was shown to be directly determined by the detectors capabilities of reconstructing secondary decay vertexes in the transverse plane, and conse-
Extraction of non-prompt $J/\psi$

quently largely dependent on the transverse momentum of the analysed tracks. A preliminary study was then performed on Monte Carlo simulated data in order to get a quantitative estimation of the effective weight of resolution effects and to consequently establish a reasonable transverse momentum working range for successive analyses on the resulting data sample. Before commenting the obtained results, it is though appropriate to spend some words on the working principles of such simulations.

**Figure 4.8** – Pseudo-proper decay length distributions of prompt $J/\psi$ extracted from Monte Carlo simulated data, for different values of applied minimum $p_T^{J/\psi}$ cuts. As transverse momentum increases, resolution improves and distributions get gradually narrower.
Extraction of non-prompt \( J/\psi \)

All simulated data were collected from Monte Carlo procedure consisting of a basic set of minimum-bias \( p - Pb \) generated events, in which a number of prompt \( J/\psi \) are injected. \( J/\psi \) are generated as un-polarized, with a transverse momentum spectrum extrapolated from previous CDF measurements [51] and according to a rapidity distribution modelled on CEM predictions. They are subsequently reconstructed, through their \( e^+e^- \) decay channel, by means of the standard ALICE experiment procedures. In particular, reconstruction procedures, performed via GEANT3 transport model, are provided of a detailed description of ALICE geometry, materials and detectors performance, so that their use can be exploited for acceptance, efficiency and resolution studies.

Figure 4.8 shows the collected results. Pseudo-proper decay length distributions of prompt \( J/\psi \) were extracted from Monte Carlo simulated data and plotted for different values of applied minimum \( p_T^{J/\psi} \) cuts. As expected, prompt \( J/\psi \) component \( x \) distribution is strongly affected by the \( p_T \) dependence of resolution effects. As the applied minimum \( p_T^{J/\psi} \) cut is increased, resolution improves and distributions get narrower, eventually resembling a Dirac Delta. A quantitative estimate of such effects is provided by Figure 4.9, where the RMS of the former distributions is plotted as function of the applied minimum \( p_T^{J/\psi} \) cut. The study actually allowed the choice of an operative \( p_T \) range to

\[
\text{Figure 4.9} \quad \text{RMS of prompt } J/\psi \text{ pseudo-proper decay length distributions from Figure 4.8 as function of applied minimum } p_T^{J/\psi} \text{ cut. Red line indicates the } p_T \text{ cut adopted for analysis.}
\]
Extraction of non-prompt $J/\psi$ be made. Simulations show an approximate saturation trend of RMS as function of the applied cuts, so that distributions may eventually be considered as nearly $p_T$ independent for high $p_T$ cut values. The total number of selected entries sample gets however drastically reduced at the same time, so that once again a compromise had to be evaluated. As depicted by the red line in the same graph, the final momentum working range for the subsequent analyses was established by applying a minimum $p_T^{J/\psi}$ cut at 1.3 GeV/c to the experimental data sample, below which simulated distributions showed an abrupt increase in RMS.

One last preliminary selection has been considered by evaluating a modification of the sample PID cuts. As discussed, data sample was provided of a standard $dE/dx$ inclusion cut of $\pm 3\sigma$ for electrons and of a $\pm 3\sigma$ exclusion cut for both protons and pions around their respective Bethe Bloch fit values. The application of such cuts is of course necessary for a correct identification of the signal component, but may be further optimized in view of the specific

![Invariant mass distribution of dielectron candidate pairs](image)

**Figure 4.10** – Invariant mass distribution of dielectron candidate pairs, plotted within the mass interval $2.4 < m^{e^+e^-} < 4.0$ GeV/c$^2$, after preliminary selection. A $p_T^{J/\psi} > 1.3$ GeV/c cut and a PID exclusion cut at $\pm 3.5\sigma$ for protons were applied. If compared to Figure 4.3, the invariant mass interval choice happens to significantly simplify the combinatorial background description.
analysis purposes as well as of the above-discussed selections. In particular, given the relatively low value of the applied minimum $p_T^{J/\psi}$ cut, data sample showed a $J/\psi$ momentum $P^{J/\psi}$ distribution peaked at about $\simeq 2$ GeV$/c$, that is, at momentum region happens to be particularly affected by proton contamination. A further increase of proton PID exclusion cut at $\pm 3.5\sigma$ was then applied for the analysed sample. Other than reducing background contamination, the applied cut proved to be particularly helpful for subsequent fittings of invariant mass distributions.

Figure 4.10 shows the invariant mass distribution candidate dielectron pairs, as analysed in the performed work, after the application of all the above-discussed cuts. If compared to Figure 4.3, a significant improvement can be noticed for what concerns both signal fraction and combinatorial background shape.

### 4.4.2 Resolution function fit

Having discussed the preliminary selections applied to the data sample, I shall explain in the following sections how the different terms appearing in equations (4.5) and (4.6) have been evaluated on the basis of the observed data.

As already introduced, the required probability density functions were firstly to be correctly parametrized, and successively to be evaluated on data by means of a series of $\chi^2$-based fits. Provided a good choice of the parametrization and an appropriate plotting of relative data, the resulting $\chi^2$ fits curve could have been considered as an estimation for the shape of the relative underlying probability functions.

The starting point will be to discuss the employed parametrization, as well as the relative fit results, for the case of prompt $J/\psi$ pseudo-proper decay length distribution, $F_{\text{prompt}}(x)$. As I am to show, the evaluation of this distribution happens to be a considerably relevant for the outcome of the whole work because its shape is strictly connected to the presence of experimental resolution effects.

The “native” distribution of prompt $J/\psi$, that is if no resolution effects were
Extraction of non-prompt \( J/\psi \) present, should be an ideal Dirac delta distribution \( \delta(x) \), centred at \( x = 0 \). Any modification observed on the experimental \( x \) distribution, may then be expressed, in much generality, by introducing a resolution density function \( R(x) \), so that:

\[
F_{\text{prompt}}(x) = \delta(x') \otimes R(x - x') = R(x) .
\]

(4.7)

\( F_{\text{prompt}}(x) \) distribution is then nothing but the effective resolution function \( R(x) \) distribution. Evaluating such a distribution allows therefore to achieve a direct estimation of the average experimental modifications relative to the pseudo-proper variable \( x \), and affecting not only prompt \( J/\psi \) distribution, but in general all the other related \( x \) distributions.

As already shown in Figure 4.8, such modifications, expressed by the convolution product with \( R(x) \), will manifest with an overall symmetrical “spreading” effect on the respective \( x \) distributions, eventually impairing the selection ability of the whole method if \( R(x) \) is too large. The preliminary minimum \( p_T^{J/\psi} \) cut was actually applied to reduce this effect.

The functional form of \( R(x) \) was parametrized as the weighted sum of two gaussian distributions, \( G_1 \) and \( G_2 \), both normalized to unity, and of a symmetric power law term \( f \propto |x|^{-\lambda} \):

\[
R(x) = w_1 \cdot G_1(x; \mu_1, \sigma_1) + w_2 \cdot G_2(x; \mu_2, \sigma_2) + w_3 \cdot f(x; \alpha, \lambda) ,
\]

(4.8)

where the two normalized gaussian distributions are given by:

\[
G(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}
\]

(4.9)

while the power law term has the stepwise form:

\[
f(x; \alpha, \lambda) = \begin{cases} 
\frac{\lambda - 1}{2\alpha\lambda} & |x| < \alpha \\
\frac{\lambda - 1}{2\alpha\lambda} |x|^{-\lambda} & |x| > \alpha 
\end{cases}
\]

(4.10)

The overall sum was normalized to unity by writing the coefficients \( w_i \) in terms of relative weights as \( w_1 = \frac{a_1}{a_1 + a_2 + a_3} \), \( w_2 = \ldots \), so that their sum resulted set equal to 1 and that the whole function verified the proprieties of a probability density.
Unfortunately, since the prompt $J/\psi$ component is unknown, there is no practical way to fit the above-defined resolution function on a set of candidates directly selected from the experimental data sample. A simulation-based approach was then employed for this step.

Prompt $J/\psi$ have been selected, according to the discussed cuts, from the same Monte Carlo simulated data described in the previous section and plotted within the $x$ range between $\pm 2000 \, \mu m$, in bins with a width of 40 $\mu m$. Range and bin width were chosen with the aim of providing a smooth distribution shape for the $\chi^2$ fit as well as a good compromise between bin content, associated statistical uncertainties and total number of bins. It should be kept in mind, however, that such a choice is practically relevant only for a better $\chi^2$ value resulting from the specific fit, and not discriminant for the outcome of the likelihood fit. The final result is indeed computed through an un-binned fitting procedure, which keeps no memory of the previously performed binned fits, and only depends on the “agreement” of the obtained shapes with the underlying probability distributions.

Figure 4.11 – Prompt $J/\psi$ pseudo-proper decay length distribution, extracted from Monte Carlo simulated data, after preliminary selection of candidates. Red line shows fit results obtained with the functional form defined in equation (4.8).
Resolution function has then been fitted on the resulting plot with the above-defined parametrization, provided of a necessary normalization constant. Figure 4.11 reports the obtained $\chi^2$ fit result. A total number of 9 parameters (2 for each gauss distribution, plus 2 for the power law term and 3 for the statistical weights) were evaluated through this fit. They were used to define the resolution function shape which was to be employed in the final likelihood fit as well as in the successive pseudo-proper distributions fits.

### 4.4.3 Invariant mass distribution fit

Even if the evaluation of the $f_B$ fraction from $J/\psi$ pseudo-proper decay length distributions can be considered as the ultimate aim of the whole work, the employed statistical procedure actually passes through the study of the measured $J/\psi$ invariant mass distribution. That is because the maximum likelihood estimation does not take account of the single $x$ distributions, but of the whole statistical mixture of $x$ and $m^{e^+e^-}$ probability functions expressed by equation (4.5). This is due to the fact that the probability of observing an $x$ value for a $J/\psi$ candidate cannot be evaluated if the overall fraction of signal candidates in the sample is unknown. Both $x$ and $m^{e^+e^-}$ projections of the likelihood function are indeed determined by the common weights $f_{Sig}$ and $(1 - f_{Sig})$ of signal and background components in the specified invariant mass window, so that it is the simultaneous observation of $x$ and $m$ for each candidate which has to be considered by the statistical procedure in order to produce any kind of evaluation.

The study of the candidate pairs total invariant mass distribution, as plotted in Figure 4.10, arises then as another fundamental step of the procedure. It may be used to fit the parametrizations of both the $M_{Sig}(m^{e^+e^-})$ and $M_{Bkg}(m^{e^+e^-})$ probability functions of equation (4.5) as well as to get a direct $\chi^2$-based estimation of the signal fraction $f_{Sig}$ within the chosen invariant mass interval ($2.4 < m^{e^+e^-} < 4.0 \text{ GeV/c}^2$). The resulting parameters will define, as known, the input functions for the likelihood fit but may also be used as a direct $\chi^2$-based reference for a quality check of the produced un-binned results.

A simultaneous fit of both $M_{Sig}(m^{e^+e^-})$ and $M_{Bkg}(m^{e^+e^-})$ on the overall mass distribution reveals however to be rather unpractical because of the large num-
Extraction of non-prompt $J/\psi$

Figure 4.12 - Invariant mass distribution of $J/\psi$ dielectron pairs, extracted from Monte Carlo simulated data, considering only true $J/\psi$ candidates. Non-prompt yield component is highlighted.

The number of parameters involved. A subdivision of the work was therefore considered for the purpose. Rather than fitting the two functions at the same time, the employed strategy consisted at first in fixing the signal $M_{\text{Sig}}(e^+e^-)$ shape parameters through a fit on a properly selected sample of signal candidates, and subsequently in fitting the remaining free parameters on the total distribution.

Figure 4.12 reports the typical signal distribution extracted from Monte Carlo simulated data, and taking into account only “true” $J/\psi$ candidates. As can be noticed, signal shape does not depend on the prompt/non-prompt origin of the candidate and is characterized by a smeared peak at the $J/\psi$ mass value $\approx 3.1\text{GeV}/c^2$ followed by a rather slowly decreasing tail at lower mass values due to electrons radiative energy losses. A commonly used parametrization for such a shape is represented by the so-called “Crystal Ball” function:

$$f(m^{e^+e^-}; \alpha, n, \bar{m}, \sigma, N) = N \cdot \begin{cases} \exp\left(-\frac{(m^{e^+e^-}-\bar{m})^2}{2\sigma^2}\right) & \text{for } \frac{m^{e^+e^-}-\bar{m}}{\sigma} > -\alpha \\ A \cdot \left(B - \frac{m^{e^+e^-}-\bar{m}}{\sigma}\right)^{-n} & \text{for } \frac{m^{e^+e^-}-\bar{m}}{\sigma} \leq -\alpha \end{cases}$$  \hspace{1cm} (4.11)
where the coefficients are given by

\[ A = \left( \frac{n}{|\alpha|} \right)^n \cdot \exp \left( -\frac{|\alpha|^2}{2} \right) \]

\[ B = \frac{n}{|\alpha|} - |\alpha| \]

while \( N \) is a normalization constant expressing the height of the peak. The function takes the same name of a SLAC electromagnetic calorimeter, in which it was introduced for the first time [53], and can be described as the composition of a gaussian distribution of mean \( \bar{m} \) and variance \( \sigma \) with a power law term defined below a certain cut step value \( \alpha \).

The simulated data of Figure 4.12 surely provide a simple way to fit the signal shape parameters, but would further constrain the obtained results to the specific Monte Carlo simulation reliability. A data-driven approach was then chosen at this step to reduce this kind of dependence.

A reasonable distribution of signal candidates was extracted by means of a background subtraction procedure. The aim was to evaluate the overall background shape of Figure 4.10 and to subtract it in order to obtain a distribution

\[ \text{Figure 4.13} \quad \text{Experimental Invariant mass distribution of J/ψ candidate pairs, extracted from the analysed data sample, after preliminary selections. Both Opposite-sign and like-sign candidates are plotted in the same range. A scaling of like-sign distribution allows a good evaluation of combinatorial background.} \]
which could be considered in principle as compatible with that of the real $J/\psi$ candidate present in the analysed sample.

The distribution of like-sign candidates, as already mentioned, proved to be of fundamental use for the purpose. As evident from Figure 4.13, the un-physical combinations of same-sign electron pairs happened to be particularly efficient in describing the combinatorial background component of the overall invariant mass distribution. A good estimation of background shape could hence be made by means of a simple scaling of the like-sign distribution.

$OS$ and $LS$ distributions were compared within the mass range $3.2 < m^{e^+e^-} < 5.0$ GeV/$c^2$, that is, within a region ideally free of signal contamination, and the best scaling factor was chosen by evaluating the integrals of the two distribution and computing the factor which minimized their difference. Results of such a procedure are reported in Figure 4.14. The computed scale factor of 1.115 was applied to like-sign distribution, which was then subtracted from the overall invariant mass distribution of Figure 4.10 in order to get the data-

Figure 4.14 – (left) Experimental invariant mass distribution of opposite-sign (OS) and like-sign (LS) $J/\psi$ candidates, selected from data sample, and plotted within the “signal-free” mass region $3.2 < m^{e^+e^-} < 5.0$ GeV/$c^2$. LS distribution has been scaled by a factor 1.115, chosen to minimize integral difference between distributions. (right) Scaled LS distribution is subtracted from OS distribution and plotted within the same range of left-panel.
based estimation of signal candidates distribution. The outcome looked much similar, within uncertainties, to the expected shape of Figure 4.12. Crystal ball function of equation (4.11) was finally fitted on such a distribution, returning the curve reported in Figure 4.15. Its parameters would have been used to define the fixed shape of the signal term $M_{Sig}(m_{e^+e^-})$ for the fitting on the overall distribution.

The function to fit at this step, was the total invariant mass probability density function, expressed by:

$$M(m_{e^+e^-}) = f_{Sig} \cdot M_{Sig}(m_{e^+e^-}) + (1 - f_{Sig}) \cdot M_{Bkg}(m_{e^+e^-})$$  \hspace{1cm} (4.12)

where the only free parameters are represented by $f_{Sig}$ plus the ones defining the background density function $M_{Bkg}(m_{e^+e^-})$.

The choice of an invariant mass working range between $2.4 < m_{e^+e^-} < 4.0 \text{ GeV}/c^2$, discussed in section 4.3.1, proved to be particularly helpful for what concerns the choice of background parametrization. As can be noticed from Figure 4.10, background component appears indeed as distributed according to a rather simple shape within the given interval. A general exponential functional form

![Figure 4.15](image)

**Figure 4.15** – Invariant mass distribution of opposite-sign candidates, subtracted of like-sign distribution scaled by a factor 1.115. Red curve represents the fit result of a crystal ball function parametrization for $M_{Sig}(m_{e^+e^-})$, defined in eq. (4.11).
Extraction of non-prompt $J/\psi$ was hence chosen as parametrization of $M_{Bkg}(m_e^+e^-)$:

$$M_{Bkg}(m_e^+e^-; \lambda, A, C) = C + A \cdot e^{-\frac{m_e^+e^-}{\lambda}}$$  \hspace{1cm} (4.13)

The full function of equation (4.12), could then be finally evaluated through a fit on the overall mass distribution, provided the constraint for the $f_{Sig}$ value to be included within the interval $[0, 1]$.

Figure 4.16 shows the results of the $\chi^2$ fit. The best-fit curve showed a nice agreement with data, and a resulting value of ?? was estimated for the fraction of signal $f_{Sig}$ within the chosen mass range. A total number of 9 parameters (5 for the crystal ball function plus 3 background parameters and $f_{Sig}$) were then extracted through the fit, among which $f_{Sig}$ could be considered as particularly useful for a cross-check with likelihood fit results.

Unlike the previous case though, the employed parametrization for equation (4.12) could not be considered as a proper probability density function, so that one further parameter had to be computed and introduced in the final

![Figure 4.16](image)

**Figure 4.16** – Overall invariant mass distribution of opposite-sign $J/\psi$ candidates, selected from data sample. Curves represent the obtained fit results for the total mass density function parametrization. Signal shape parameters were fixed from a fit on the OS – LS distribution, background was parametrized with a generic exponential form. The reported value of $f_{Sig}$ is the $\chi^2$-based estimation of the overall signal fraction resulting from this fit.
likelihood fit in order set the overall function integral equal to unity.

### 4.4.4 Pseudo-proper background fit

The employed strategy of estimating likelihood function parameters through binned fits on observed data, probably makes the evaluation of pseudo-proper background density function, $F_{Bkg}(x)$, the most challenging step of the whole procedure. The employed parametrization must account correctly of resolution effects affecting any $x$ distribution and a proper set of candidates must be selected out of the whole sample to reproduce the, mostly unknown, features of the expected distribution for fitting. Contrarily to the invariant mass distribution case, now there is no practical way to single out background component from the overall candidates $x$ distribution without the risk of significantly affecting the unknown non-prompt component. Moreover, an effective description from first principle of the underlying distribution appears as infeasible because the existence of background sources of unknown origin cannot be excluded. Any kind of estimation at this step must therefore rely on a number of reasonable, but still arbitrary, assumptions.

Perhaps the most natural way to carry out a selection of reasonable candidates for the fitting of $F_{Bkg}(x)$ consists in choosing a set of dielectron pairs whose invariant mass value $m^{e^+e^-}$ falls out of the most probable signal region. That is, in practice, either in the left ($m^{e^+e^-} \lesssim 2.6 \text{ GeV/c}^2$) or in the right ($m^{e^+e^-} \gtrsim 3.2 \text{ GeV/c}^2$) invariant mass window around the measured signal peak. The main uncertainty affecting this rather simple approach lies though in the intrinsic arbitrariness of the chosen interval, as well as in the assumption that the resulting candidates distribution provide a agreement with the actual background function. The outcome, moreover, may also depend on whether the left or right band is chosen for selection as well as on the actual chosen band width. For these reasons, a preliminary study was necessary to evaluate the possible implications arising from the selection of a specific set of background candidates.

A first qualitative estimation was made by comparing the different distributions resulting from the choice of one of the two mass bands. In Fig-
Extraction of non-prompt $J/\psi$ candidates extracted from both the left mass band in $2.4 < m^{e^+e^-} < 2.6\text{GeV}/c^2$ and the right band in $3.2 < m^{e^+e^-} < 4.0\text{GeV}/c^2$ are plotted over the same $x$ range, for different values of minimum $p_T^{J/\psi}$ cut. Despite the different statistics of the resulting samples, results does not exhibit a significant difference in shapes between the two bands distributions, and not even a particular dependence on the $p_T^{J/\psi}$ cut. All the different samples seem to describe the same characteristic shape, with a smeared central peak and a nearly flat outer distribution of random single entries extending over high

Figure 4.17 – Pseudo-proper decay length distributions of opposite-sign $J/\psi$ candidates from both the “left-band” ($2.4 < m^{e^+e^-} < 2.6\text{GeV}/c^2$) (green markers) and the “right-band” ($3.2 < m^{e^+e^-} < 4.0\text{GeV}/c^2$) (blue markers) invariant mass regions, plotted for different values of minimum $p_T^{J/\psi}$ cut. Despite the lower statistics of the narrower left-band sample, the distributions show a significant agreement in their shapes regardless of the applied cut.
Figure 4.18 – RMS of pseudo-proper decay length distributions of Figure 4.17 is plotted as function of applied \( p_T \) cut, together with the “side-bands” \((2.4 < m^{e^+e^-} < 2.6 \text{ GeV/c}^2 \cup 3.2 < m^{e^+e^-} < 4.0 \text{ GeV/c}^2)\) and like-sign distributions. All distributions show a low dependence on \( p_T \), but like-sign appear as more widely spread than opposite-sign candidates.

\( x \) values. A more quantitative insight is though provided by Figure 4.18, in which, in a similar way as what done for the study of resolution effects, the previous shapes are evaluated in terms of RMS and plotted as function of applied \( p_T^{J/\psi} \). Results of the same study performed on the “side-bands” distribution \((2.4 < m^{e^+e^-} < 2.6 \text{ GeV/c}^2 \cup 3.2 < m^{e^+e^-} < 4.0 \text{ GeV/c}^2)\), for both opposite-sign and like-sign pairs, are also reported in the same graph.

This time, like-sign distribution clearly do not provide a meaningful tool for the evaluation of \( x \) background. While left and right band candidates distribute according to similar shapes with nearly the same value of RMS, like-sign candidates show a much larger distribution, with an RMS value almost way over the one of opposite-sign pairs. Both the chosen left and right band distributions show, furthermore, that their RMS is nearly independent from the applied \( p_T \) cut, which is actually good signature of their reliability.

The same study was repeated for different choices of mass windows, band widths as well as for different \( x \) plotting ranges. Results happened to be in qualitative agreement with the exposed picture, and seemed to show how most of the discrepancies between the RMS of two bands, especially at higher \( p_T \) values, were only due to the statistical fluctuations in the flat region with single entries over high \( x \) values.
The definitive range for background evaluation was finally chosen to be the above-mentioned side-bands region: $2.4 < m^{e^+e^-} < 2.6\text{GeV}/c^2 \cup 3.2 < m^{e^+e^-} < 4.0\text{GeV}/c^2$. With respects to the single left and right bands, their union provides in facts not only a larger number of candidates, essential for a better fitting of the complex background shape, but also an average shaping of their two resulting distributions. Actually, sample statistics could have been further increased by enlarging the side bands over the specified mass working range of $2.4 < m^{e^+e^-} < 4.2\text{GeV}/c^2$. This, however, would have resulted in a risky choice since, as explained, any $J/\psi$ candidate outside that range would not have been considered for the final likelihood estimation.

Assuming the chosen sample as representative of the background component, the probability function $F_{\text{Bkg}}(x)$ could finally be fitted on the resulting distribution. The chosen parametrization for $F_{\text{Bkg}}(x)$ was the same one used in previous $p-p$ analysis [52] [51]. It proved to be efficient describing the peculiar shape of the side-bands distribution, as well as to take into account resolution effects. Its functional form is given by:

$$F_{\text{Bkg}}(x) = (1 - f_+ - f_- - f_{\text{sym}}) \cdot R(x) +$$
$$+ \left( \frac{f_+}{\lambda_+} e^{-\frac{x'}{\lambda_+}} \cdot \theta(x') + \frac{f_-}{\lambda_-} e^{\frac{x'}{\lambda_-}} \cdot \theta(-x') + \frac{f_{\text{sym}}}{2\lambda_{\text{sym}}} e^{-\frac{|x'|}{\lambda_{\text{sym}}}} \right) \otimes R(x - x')$$

(4.14)

where $\theta(x)$ is the step function and $R(x)$ is the previously computed resolution function.

The parametrization has a rather complex form, in which the first term $\propto R(x)$ is a pure resolution term describing the residual combinatorics of prompt particles, while the other terms, all convoluted with the resolution function, represent exponential terms for the description of the symmetric central peak ($\propto e^{-\frac{|x'|}{\lambda_{\text{sym}}}}$) and of the slowly decreasing negative ($\propto e^{\frac{x'}{\lambda_-}}$) and positive ($\propto e^{\frac{x'}{\lambda_+}}$) outer tails. The coefficients $f_-$, $f_+$ and $f_{\text{sym}}$ represent finally the statistical weights for each of such terms. The introduction of all these components is needed in order to account for possible asymmetries arising from random combinations of electrons from semi-leptonic decays of charm and beauty hadrons, which tend to produce positive x values, as well as of other secondary or mis-reconstructed tracks which contribute both to positive and negative x values.
Each term is by definition normalized to unity and the sum of the weights was set equal to 1 in order to ensure that the whole function $F_{Bkg}(x)$ verified the proprieties of a probability density function. Other more complex parametrizations have actually been proposed and considered for the work but they resulted in negligible differences in the final likelihood fit.

The results of the fit with the above-defined expression for $F_{Bkg}(x)$ on the side-bands candidates distribution are reported in Figure 4.19, together with the curves representing the single terms of equation (4.14). A total number of 7 effective parameters (3 for the exponential terms constants plus 3 for their respective weight and a further weight for the resolution term) were computed through this fit. These could have been employed as fixed, or in part as free parameters for the final likelihood fit.

One final remark should though be mentioned for what concerns the adopted fitting procedure for the background function. Contrarily to all the previous cases, a $\chi^2$-minimization fit didn’t prove to be an effective way to evaluate a

![Figure 4.19](image.png)

**Figure 4.19** – Pseudo-proper decay length distribution of $J/\psi$ candidates from the side-bands mass region ($2.4 < m^{+}e^- < 2.6 \text{ GeV}/c^2$ $\cup$ $3.2 < m^{+}e^- < 4.0 \text{ GeV}/c^2$), fitted with the background parametrization of equation (4.14). Black curve is the result of a likelihood-fit. Coloured curves represent the different terms appearing in the parametrization of $F_{Bkg}(x)$. 
reasonable probability density function from the chosen distribution. $\chi^2$-based fits, in facts, typically returned curves with excessively flat exponential tails, leading to un-meaningful results when interpreted as probability densities from the likelihood minimization. The reason is that, as already mentioned, when very low-statistics are involved, as is the case of the high-$x$ regions of the background distributions, $\chi^2$ best-fit curves provide no longer a good approximation of the underlying probability density functions. A likelihood-minimization fit, based on the same method introduced in section 4.3, was then found to be the most suited choice for the purpose.

4.4.5 Non-prompt $J/\psi$ template function

The last term required for the maximum likelihood estimation is the $F_B(x)$ function of equation (4.6), representing the pseudo-proper decay length probability distribution of $J/\psi$ from b-hadrons decays. Since the non-prompt component of the $J/\psi$ yield is unknown, this time there is no effective way to fix the shape of $F_B(x)$ through fits on experimental distributions. A simulation-based approach is therefore once again required for the purpose.

It must be considered that the $F_B(x)$ function parametrizes the actual $x$ distribution of reconstructed non-prompt $J/\psi$ candidates as measured in the experiment, which means that it must account not only of the native non-prompt $J/\psi$ kinematic distribution, but also of experimental resolution effects arising from tracks reconstruction.

In general terms, it is then possible to express $F_B(x)$ as

$$F_B(x) = \chi_B(x)' \otimes R(x - x')$$  \hspace{1cm} (4.15)$$

that is, as the convolution of the native $x$ distribution $\chi_B(x)'$ arising from pure kinematic of b-hadron decays, with the effective resolution density function $R(x)$. Resolution function parameters were already fixed from the previous fits, so that only the kinematic part, $\chi_B(x)'$, is to be computed.

Monte Carlo simulations luckily provide a well-established way to compute such distribution with a certain degree of reliability. For the specific case, the distribution was computed on the basis of simulated events generated by PYTHIA event generator, set to produce a $b\bar{b}$ quark pair for each $p-p$ collision.
event. Both quarks hadronize in b-hadrons, which are subsequently forced to
decay into a $J/\psi$ state, within the central rapidity region.
The simulated distribution of non-prompt $J/\psi$ extracted from this kind of
computation, without considering any effect due to experimental resolution,
is reported in Figure 4.20. The Figure actually reports the $\chi_B(x)'$ distribu-
tion for the relevant case non-prompt $J/\psi$ with $p_T^{J/\psi} > 1.3$ GeV/c which was
employed for the maximum likelihood minimization. Distributions of Figure
4.6, reported in section 4.2, can be considered as the resulting distributions
originated from such templates when resolution effects are included.

Given all the necessary parametrizations for the likelihood function terms,
the likelihood maximization procedure can finally be computed on the basis
of the selected sample. The following chapter will be dedicated to comment
the maximum likelihood outcome, as well as to the description of the possible
uncertainties affecting the obtained results.
$p_T$-integrated measurement of non-prompt $J/\psi$

The extraction of the so-called non-prompt component of $J/\psi$ yields, that is, of the overall fraction of $J/\psi$ resulting from the decay of beauty flavoured hadrons, arose in the past decade as an important aim of inclusive $J/\psi$ analyses. Its measurement not only allows the study of prompt $J/\psi$ production cross sections, which is of greater relevance for the testing of charmonium production models as well as of several predictions of Quantum Chromodynamics, but is also directly linked to beauty-flavoured hadrons production, which constitute a main object of study for heavy-flavour physics and have proven to be valuable tools for the investigation of heavy-ion collisions and related phenomena.

Up to now, LHC experiments have managed to separate the prompt and non-prompt component of $J/\psi$ yield at mid-rapidities from $p-p$ collisions collected during the 2010 runs at $\sqrt{s} = 7$ TeV, providing quality results over a wide transverse momentum range (Figure 5.1). These measurements will constitute the main baseline for present analyses on heavier $Pb-Pb$ collisions, and ALICE experiment, in particular, was qualified for carrying out the measurements in the low transverse momentum region.

The analysis reported during this thesis work, fits in this context as a natural extension of the former $p-p$ analyses to the newly collected data from $p-Pb$ collisions, considered as “benchmark” experiments for the proper interpreta-
tion of many features of heavy-ion collisions. Purpose of the work, has been to evaluate the overall non-prompt \( J/\psi \) fraction in a sample inclusive \( J/\psi \) candidates reconstructed from their electromagnetic decay channel \( J/\psi \rightarrow e^+e^- \) among a collection of minimum-bias \( p-Pb \) collisions at the c.m.s. energy per nucleon pair of \( \sqrt{s_{NN}} = 5.02 \) TeV in the central rapidity region \( |y_{J/\psi}| < 0.9 \) of the ALICE experiment at LHC.

In the previous chapter I have reported in detail how such a task has been carried out on the analysed data sample by means of a well-defined statistical procedure operating on the measured distributions with respect to the “pseudo-proper decay length”, a properly defined variable which proved to be capable of distinguishing between the prompt and non-prompt components of the sample. The employed procedure was able to extract a meaningful value for the non-prompt fraction, \( f_B \), through an un-binned maximum likelihood estimation, provided that a proper parametrization of the required probability density functions of the sample were given.

In order to reduce uncertainties arising from the large number of parameters involved, the work has been subdivided by fixing most of the required func-

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**Figure 5.1** - Fraction of \( J/\psi \) yield resulting from \( b \)-hadrons decays, as measured by CDF (left) in high-statistics \( pp \) collisions at \( \sqrt{s} = 1.96 \) TeV [51], and by LHC experiments (right) in \( pp \) collisions at \( \sqrt{s} = 7 \) TeV as function of \( J/\psi \) \( p_T \). ALICE’s results [52] complement CMS, ATLAS and LHCb measurements in the low transverse momentum region.
\( p_T \)-integrated measurement of non-prompt \( J/\psi \)

The application of maximum likelihood estimation method allowed a \( p_T \)-integrated measurement of the overall non-prompt fraction in a sample of inclusive \( J/\psi \) candidates collected during the runs of \( p - Pb \) collisions at \( \sqrt{s_{NN}} = 5.02 \) TeV performed in the first months of 2013 at ALICE experiment.

Multiple fits, performed on properly selected distributions, allowed the evaluation of the almost 30 parameters needed to describe the different probability density functions required for the computation of the likelihood function \( F(x, m^{e^+e^-}) \). In particular, the invariant mass total distribution \( M(m^{e^+e^-}) \) and the pseudo-proper decay length background distribution \( F_{\text{Bkg}}(x) \) were directly evaluated on the basis of the analysed sample, whereas the prompt and non-prompt distributions, \( F_{\text{prompt}}(x) \) and \( F_{\text{B}}(x) \), required the use of Monte Carlo simulated data.

All the above computed parameters were set as fixed values for the likelihood fit, with the only exception of \( f_B \) and \( f_{\text{Sig}} \), representing the overall fractions of non-prompt and signal \( J/\psi \) candidates within the chosen invariant mass range of \( 2.4 < m^{e^+e^-} < 4.0 \text{GeV}/c^2 \). The likelihood fit was then performed in order to get a \( p_T \)-integrated estimate of the non-prompt fraction.

This last chapter will be dedicated to the description of the results returned from the maximization procedure. The first section will report the projection of the maximized likelihood function on both the invariant mass and pseudo-proper decay length axis, along with the computed value of the non-prompt fraction \( f_B \). In the second section, the approach followed to get a first estimation of the systematic uncertainties affecting the obtained results will be described, whereas the third section will be finally devoted to explain possible future outlooks and development.

### 5.1 Likelihood Fit Results

The application of maximum likelihood estimation method allowed a \( p_T \)-integrated measurement of the overall non-prompt fraction in a sample of inclusive \( J/\psi \) candidates collected during the runs of \( p - Pb \) collisions at \( \sqrt{s_{NN}} = 5.02 \) TeV performed in the first months of 2013 at ALICE experiment.

Multiple fits, performed on properly selected distributions, allowed the evaluation of the almost 30 parameters needed to describe the different probability density functions required for the computation of the likelihood function \( F(x, m^{e^+e^-}) \). In particular, the invariant mass total distribution \( M(m^{e^+e^-}) \) and the pseudo-proper decay length background distribution \( F_{\text{Bkg}}(x) \) were directly evaluated on the basis of the analysed sample, whereas the prompt and non-prompt distributions, \( F_{\text{prompt}}(x) \) and \( F_{\text{B}}(x) \), required the use of Monte Carlo simulated data.

All the above computed parameters were set as fixed values for the likelihood fit, with the only exception of \( f_B \) and \( f_{\text{Sig}} \), representing the overall fractions of non-prompt and signal \( J/\psi \) candidates within the chosen invariant mass range of \( 2.4 < m^{e^+e^-} < 4.0 \text{GeV}/c^2 \). The likelihood fit was then performed in order to get a \( p_T \)-integrated estimate of the non-prompt fraction.

This last chapter will be dedicated to the description of the results returned from the maximization procedure. The first section will report the projection of the maximized likelihood function on both the invariant mass and pseudo-proper decay length axis, along with the computed value of the non-prompt fraction \( f_B \). In the second section, the approach followed to get a first estimation of the systematic uncertainties affecting the obtained results will be described, whereas the third section will be finally devoted to explain possible future outlooks and development.
**Figure 5.2** — Projection of the maximized likelihood function on the pseudo-proper decay length axis. The different terms of equations (4.5) and (4.6) are plotted scaled by the number of measured entries multiplied by bins width. In particular, blue curve represents the total likelihood result, red curve stands for prompt $J/\psi$, purple for non-prompt $J/\psi$ and grey for background.

The working range of $2.4 < m^{e^+e^-} < 4.0$ GeV/c$^2$.

The final results of the whole maximization procedure are reported in Figures 5.2 and 5.3, where the two different projection of the maximized likelihood function, on both the $x$ and $m^{e^+e^-}$ axis respectively, are plotted upon the experimentally observed distributions. Likelihood fit actually returns only the shapes considered as the most likely approximation of the sample underlying probability density functions. In order to match the observed data, such shapes were then simply scaled by the number of measured entries multiplied by bins width, with a resulting remarkable agreement. The different terms constituting the overall likelihood function, scaled by their respective computed statistical weight, are also plotted on the same distribution.

The resulting final estimate of the value of non-prompt fraction in the analysed sample has been found to be

$$f_B = 0.138 \pm 0.040$$
where the error is referred to the statistical uncertainties arising from the application of the likelihood method. Also the fraction of signal candidates within the mass range $2.4 < m^{e^+e^-} < 4.0$ GeV/$c^2$ was estimated, returning a value of:

$$f_{\text{Sig}} = 0.218 \pm 0.015.$$  

The fact that both $x$ and $m^{e^+e^-}$ likelihood function projections, evaluated by means of an un-binned fitting procedure, simultaneously provide a remarkable agreement with the binned distributions is actually a sign of the successful outcome of the statistical procedure. The previous binned fit estimate of $f_{\text{Sig}}$ results within uncertainties compatible with the likelihood-based value, while the estimate of $f_B$ seems to agree with the trend of the past $p-p$ measurements in the low $p_T$ region. All of these observations may be considered as signatures for the reliability of the obtained results.

**Figure 5.3** - Projection of the maximized likelihood function on the invariant mass axis. The terms of equation (4.12) are plotted scaled by the number of measured entries multiplied by bins width. In particular, blue signal curve represent the total signal component, whereas green curve only the non-prompt fraction.
5.2 Checks on Systematics

The errors reported in the previous section for the values of $f_B$ and $f_{Sig}$ represent only the statistical uncertainties resulting from the likelihood maximization procedure. A more detailed estimation should however account also of possible contributions which may systematically shift the actual value of the likelihood function.

In order to get a first estimate of the systematic uncertainties affecting the obtained value for $f_B$, a number of checks has then been performed considering their most significant sources. In particular, the contributions arising from the evaluation of $R(x)$, $M(m^{e^+e^-})$ and $F_{Bkg}(x)$ terms in the likelihood function have been considered. In the context of this un-comprehensive study, the effective impact of the considered contributions on the final result has reasonably been over-estimated to account for other not considered sources of uncertainty.

Resolution function.

The resolution function term, $R(x)$, happens to affect in a large way the evaluation of the likelihood function. It appears as a convolution factor in all of the considered pseudo-proper decay length distributions, and may in principle even impair the separation capability of the whole method if it happens to be too large.

Its evaluation has been performed, as described in section 4.4.2, through the fit of prompt $J/\psi$ distribution extracted from simulated data. It is hence natural to assume a form of intrinsic systematic uncertainty in all the performed calculations arising from the non-correct estimations of the actual experimental resolution effects by the employed Monte Carlo generator.

A quantitative estimation of such a contribution to the likelihood function results has been performed by repeating the maximum likelihood fit with an artificially different shape for the input resolution function.

The RMS of $R(x)$ was varied by modifying the resolution formula according to the relation:

$$R'(x) = \frac{1}{1 + \delta} \ R\left( \frac{x}{1 + \delta} \right),$$

(5.1)
in which the $\delta$ parameter easily allows to set the desired fractional variation of the function RMS. 
The fit was then repeated for the two reasonably extreme cases of $\pm 10\%$ variation in resolution RMS. The differences with the original value of $f_B$ were considered as the systematic uncertainties resulting from resolution contribution.

**Invariant mass.**

A further contribution to the systematics of $f_B$ has been considered in the performed evaluation of the total invariant mass distribution parameters. The $M(e^+e^-)$ term of the likelihood function was computed by fixing the signal shape parameters through a background subtraction procedure estimated on the basis of like-sign candidates invariant mass distribution. In order to get a first estimate of the systematics resulting from such a data-driven approach, a different signal shape extraction was considered. 

Signal function parameters were then fixed by fitting the Monte Carlo invariant mass distribution of signal candidates of Figure 4.12 and the total invariant mass function fit was repeated with the new resulting signal shape. Likelihood fit was hence repeated with the newly computed $M(e^+e^-)$ parameters. Once again, the difference with the original value of $f_B$ was considered as the relevant systematic uncertainty.

**Pseudo-proper decay length background.**

One last contribution to $f_B$ systematic uncertainties was evaluated by considering the pseudo-proper decay length background distribution, $F_{Bkg}(x)$. For the estimation of its parameters, a fit was performed on the distribution of $J/\psi$ candidate with invariant mass falling within the “side-bands” region $(2.4 < m^{e^+e^-} < 2.6 \text{ GeV}/c^2 \cup 3.2 < m^{e^+e^-} < 4.0 \text{ GeV}/c^2)$. Such an arbitrary choice may though result in rather different background shapes for the likelihood function if other set of candidates were chosen for the fit. 

To get an estimation of the systematic uncertainty arising from this kind of possibility, likelihood was repeated by changing the input background shapes. In particular, two extreme cases were by setting a simultaneous variation of all
the background exponential parameters ($\lambda_-, \lambda_+, \lambda_{sym}$) of $\pm 10\%$ with respect to their former values. Differences with the original value of $f_B$ were considered as the background systematics.

All the computed obtained results are summarized in the following table.

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Systematics (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resolution Function</td>
<td>$-0.20 + 0.22$</td>
</tr>
<tr>
<td>Invariant Mass Distribution</td>
<td>$\pm 0.24$</td>
</tr>
<tr>
<td>x Background Distribution</td>
<td>$-0.12 + 0.10$</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>$-0.33 + 0.34$</td>
</tr>
</tbody>
</table>

The final measurement of non-prompt $J/\psi$ fraction, may now be more significantly expressed as:

$$f_B = 0.138 \pm 0.040 \ (stat.) ^{+0.34}_{-0.33} \ (syst.)$$

where the last error is the one resulting from such an estimate of systematic uncertainties, assumed as root sum squared of the three considered contributions.
Conclusions

The value of $f_B$ extracted through the employed maximum likelihood estimation method, may be considered as a first quantitative estimate of the actual $p_T$-integrated non-prompt component of inclusive $J/\psi$ yields in $p−Pb$ collisions at LHC energies. The good agreement of likelihood functions with experimental distributions, as well as the accordance with previous $p−p$ measurements are surely hints for the quality of the obtained result.

A number of further studies may though be performed to improve the reliability of the reported result. Much of them naturally concern features which were not exhaustively taken into account in this thesis work. One first remark can be made for what concerns acceptance and efficiency corrections. The value returned from the employed statistical procedure represents the measured fraction of reconstructed $J/\psi$ resulting from b-hadrons decays, which may in principle not be coincident with the actual fraction of $J/\psi \leftarrow B$. It does not consider, in facts, that acceptance and efficiency corrections may in principle be different for prompt and non-prompt $J/\psi$, because either different $p_T$ distributions or different average polarizations may influence the resulting corrections. Even if it consists in a reasonably small corrections for $p_T$ integrated measurements, an effect of this kind should surely be evaluated for a more precise evaluation.

Systematic uncertainties may also be more carefully evaluated by taking into account of other not considered contributions. Among them is, for example, the estimation of the systematic uncertainties introduced in the reconstruction of primary vertex position. Being the non-prompt component a non negligible fraction of total $J/\psi$ yield, a slight bias to vertex position could indeed be expected due to the average displacement caused by non-prompt candidates tracks. A quantitative estimation may then be made by recalculating primary
vertex position without accounting of any \( J/\psi \) tracks. Polarization effects represent another source of unconsidered systematics. That is because all the performed evaluations based on Monte Carlo simulated data, assume prompt \( J/\psi \) as produced un-polarized. This would result, as mentioned, in different acceptance and efficiency corrections. In view of the above-discussed perspectives, the value of \( f_B \) reported as result of this thesis work may be considered as an effective preliminary baseline for future and more detailed incoming analyses.
Bibliography


113


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