Infrared and terahertz imaging through self-mixing interferometry in quantum cascade lasers

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# Summary

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Introduction

Best known for the development of the elegant mathematical formulation of the electromagnetic theory and the partial differential equations named after him, the Scottish physicist James Clerk-Maxwell also dabbled in photography throughout his life. One of his lesser-known contributions lies in the field of colour vision and consists in the propose of the first permanent colour image, obtained overlapping three photographic plates taken with a red, blue, and green colour filters (Fig. 1)\textsuperscript{1}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{maxwell_color_photo.png}
\caption{The first permanent color photograph made with the three-color method suggested by James Clerk Maxwell in 1856 and experimentally taken in 1861 by Thomas Sutton. The subject was a tartan ribbon tied into a bow. The photographic plates are exposed in Maxwell’s house, which became a museum in Scotland.}
\end{figure}

Like him, many scientists have been enthralled by imaging science, and during the last century a huge number of schemes has emerged, exploiting photons which belong not only to the tiny visible slice of the electromagnetic spectrum.

Up-to-date real-world applications make use of radar false-color images of clouds for weather forecast, x-ray pictures of broken bones or baggage content for security screening, thermal images for military use on the battlefield, high-resolution infrared photographs of the earth taken from satellite orbits, and many other categories too numerous to be listed here. Different wavelengths and techniques are suited for different purposes in every-day life and in a motley onset of research fields.

Notwithstanding, during the last thirty years, the terahertz (THz) range, i.e. the electromagnetic region sandwiched between mid-infrared and microwaves \((0.1-10 \ \text{THz, } 3\text{cm}^{-1}-330\text{cm}^{-1}, \ 30 \ \mu \text{m}-3\text{mm})\), has attracted attention because of its unique interaction proprieties with matter. Indeed, the so-call T-rays can pass through materials that appear opaque at visible wavelengths, such as ceramic, clothes, plastics and packaging, while being reflected from metals. These features, combined with the fact that most spectroscopic signatures of substances of great interest - drugs and explosives first of all - lie in the terahertz regime, have led to the flourish of THz imaging science and THz systems used for multidisciplinary non-destructive evaluations. Besides, thanks to its non-ionizing energy and the peculiar absorption from water molecules, this radiation has been exploited in medical diagnosis, in screen for skin cancers and tooth decays too.
However, up to now both the lack of compact sources and detectors and the high THz absorption of atmosphere has still limited the use of THz radiation in laboratory. For these reasons, this region has been considered a terra nullius where both photonics and electronics have tried to face these problems from two opposite sides, with different schemes and approaches.

Under these circumstances, one of the most promising devices coming from the photonics branch is the quantum cascade laser (QCL). Through this system - demonstrated in 1994 in mid-Infrared and re-invented in 2001 to work in THz regime - it is possible to design optical transitions in the ranges of 1-5 THz and 15-100 THz, making use of the principles of quantum mechanics.

This thesis discusses the implementation of an imaging system, which makes use of a THz and Mid-Infrared QCL in self-mixing configuration. This coherent scheme, in which a laser is used both as source and detector, exploits the interference between the electric field inside the cavity laser and the back-coupled radiation reflected or scattered from an external target, in order to get the image of a sample.

This text is organized as follows.

**Chapter 1** reviews THz imaging techniques with pulse and continuous sources, giving some details about near field approaches.

**Chapter 2** discusses optical feedback in quantum cascade lasers, presenting the working principles of self-mixing interferometry in conventional laser and comparing differences with the QCL dynamics.

**Chapter 3** describes mid-infrared and THz setups, with a focus on some experimental critical aspects.
Chapter 4 illustrates and discusses imaging results with different samples, drafting some experimental evidences about the dynamics of QCL subjected to optical feedback.

Chapter 5 analyses preliminary results and highlights the opportunities to extend this scheme in order to acquire the charge-distribution of a semiconductor device.
Chapter 1

Terahertz imaging review

In this chapter is presented an overview of active imaging techniques in the terahertz domain. After discussing some of the features of this unexplored spectral region (1.1), imaging schemes which make use of broadband and continuous (CW) sources are respectively reviewed in paragraph 1.2 and 1.5. In addition, near field methods and tomographic techniques are sketched in paragraph 1.3 and 1.4. We conclude the chapter outlining a series of applications for imaging terahertz that involves a vast number of different research areas (paragraph 1.6).

1.1 The terahertz gap

The terahertz (THz) region lies in the frequency range from 0.1 to 10 THz and exists between two readily developed frequency bands: the microwave and the infrared. This so-called terahertz gap has historically been defined by the relative lack of well-established sources and detectors and is therefore explored to a lesser extent.

While in the microwave regime, electromagnetic radiation is typically generated through high-frequency oscillating charges and the photon energy involved is smaller than thermal energy $k_B T$, in the infrared range
1.1 The terahertz gap

geneneration mechanisms employ optical transitions between quantized energy levels in a semiconductor. The terahertz domain is therefore the natural bridge between the classical and quantum mechanical descriptions of electromagnetic waves and their interactions with matter. Much of the advances in terahertz science and technology have emerged from the overlap originated from these two different points of view, borrowing ideas from each.

Although terahertz technology is still in its infancy, this region has been investigated since 1970s by space scientists. The reason resides in the fact that more than 98 percent of the photons released since the Big Bang, which give information about chemical compositions of the interstellar medium and planetary atmospheres, fall in the sub-millimeter and far-infrared bands.

Moreover, many molecules that constitute the atmosphere exhibit their rotovibrational spectra in the THz region. Thus, the so-called T-rays are suited as spectroscopic tools for pollution monitoring.

On the other hand, every long-distance application that has been suggested in the last years, had to come up against the downside of atmospheric absorption (Figure 1.1), which can reach some decibel (dB) per meter at standard humidity conditions. In this direction, the development of source with a narrowed emission spectrum is a crucial aspect.
1.1 The terahertz gap

**Figure 1.1** Signal attenuation in the range of 0.5-5 THz caused by atmospheric absorption at normal humidity from water vapor and oxygen. Taken from HITRAN database.

A great number of devices have been proposed as terahertz sources, each of those employing a characteristics physical mechanism to generate radiation.

**Figure 1.2** Sources output power versus frequency. Adapted from [1]
Figure 1.2 tries to summarize them, depending on the average emission power as a function of frequency. Sources fall into three broad categories: solid state (counting harmonic frequency multipliers, transistors and monolithic microwave integrated circuits), vacuum (including backward-wave oscillators, grating-vacuum devices, klystrons, traveling-wave tubes, and gyrotrons), and laser and photonic (including quantum cascade lasers, optically pumped molecular lasers and a variety of optoelectronic radio frequency generators).

Quantum cascade laser, above all, is the most promising compact source for frequencies higher than 1 THz, nevertheless up to now it requires cryogenic cooling. We will describe its working principles in details in paragraph 2.5.
1.2 Terahertz time-domain spectroscopy

![Figure 1.3 Typical schematic for terahertz time-domain spectroscopy. Adapted from [2]](image)

During the last twenty years most research developments in terahertz sensing and imaging have made use of terahertz time-domain spectroscopy (THz-TDS), first introduced by van Exter in 1989 [3]. The basic general TDS experimental setup is based on a pump-probe scheme (Figure 1.3). Here, femtosecond laser produces an infrared pulse train at a repetition rate usually near 100 MHz, which is separated in a pump and a probe beam by means of a beam splitter. The former arrives at the emitter and gives rise to a sub-picosecond THz pulse, which travels through the sample and is focused onto an ultra-fast detector. The THz-induced transients in the detector are measured by the probe pulse, which is delayed by a mechanical delay line. In this way, the amplitude $E(t)$ of the THz electric-field is detected by temporally sampling the waveform using the probe pulse.

The two most popular mechanisms for the generation and the detection of THz broadband fields involve transient photocurrents in photoconductive
antennas and second-order effects in nonlinear optical crystals. We will give a qualitative description of these processes below.

Photoconductive techniques make use of the optical resonant excitation of a semiconductor to generate currents through the production of electrons–holes pairs (see Figure 1.4 (a)). A metal pattern is typically deposited on a semiconductor substrate, in the form of an H-like antenna structure. A static bias applied across the electrodes accelerates the free carriers that are optically generated by the femtosecond-laser and, simultaneously, the charge density decays mainly by trapping in defect sites. According to the Maxwell’s equations, the impulse current arising from this acceleration and decay of free carriers is the source of a subpicosecond pulse of electromagnetic radiation \( E(t) \sim \partial J/\partial t \), which contains frequency components from approximately 0.1 to 5 THz. The antenna can be modeled as a hertzian dipole and its design also helps to couple the THz radiation into free space.

The underlying physics of photoconductive detection is almost the same as the generation method described above (see Figure 1.4 (b)). Again, the infrared beam is focused at the center of an antenna, generating hole electrons pairs. When the THz pulse arrives during the lifetime of the carriers they are accelerated towards the electrodes. Although this short current pulse is too fast to be resolved using conventional electronics, the average current (averaged over many identical pulses) can be measured as a function of the delay between the optical gate pulse and the THz pulse. Fast photoconductors such as radiation-damaged silicon-on-sapphire (SoS), low-temperature-grown gallium arsenide (LT-GaAs) or indium phosphide (InP) are used to provide sub-picosecond sampling resolution.
Figure 1.4. Photoconductive emitter (a) and photoconductive detector antenna (b) mounted on a hemispherical lens. (c) Scheme for free-space EO sampling: probe polarizations with and without a THz field are shown before and after the polarization chain. From [4]

The second class of schemes employed to generate and detect broadband THz radiation exploits respectively optical rectification and Pockels effect exhibited by second-order nonlinear crystals.

When an optical pulse of frequency $\omega$ and duration $t_p$, $E(t) = E_0 e^{-(t/t_p)^2} e^{-i\omega t}$ interacts with a non-centrosymmetric medium, such as Zinc telluride (ZnTe), the nonlinear polarization induced by the optical rectification replicates the pulse envelope $P(t) = P_0 e^{-(t/t_p)^2}$. This generation process can be interpreted as the difference mixing between all possible pairs of spectral components within the bandwidth of the fs-optical pulse and, using the nonlinear optics formalism, we can write the polarization as

$$P_i^{(2)}(0) = \sum_{j,k} \varepsilon_0 \chi^{(2)}_{ijk}(0,\omega,-\omega) E_j(\omega) E_k^*(\omega),$$

where $i,j,k$ are the cartesian indexes and $\chi^{(2)}_{ijk}$ is the second-order nonlinear susceptibility tensor. Neglecting walk-off effects, the polarization acts as
the source of the electromagnetic field. Thus a broadband terahertz field is generated, whose bandwidth is roughly the inverse of the optical pulse duration $t_p$.

THz detection via nonlinear effects is carried out through the Pockels effect: while the linearly polarized optical pulse propagates through the electro-optic crystal, the THz field induces a birefringence in the medium, resulting in an elliptical polarization of the probe beam. After passing through a $\lambda/4$-plate the probe beam is split in two orthogonal polarized components via a Wollaston prism. A balanced photo-detector measures the intensity difference between these two components, which is proportional to the THz field amplitude.

![Graphs of time-domain terahertz waveform and their Fourier-transformed spectra.](image)

**Figure 1.5** (left) Examples of time-domain terahertz waveform emitted from p-InAs photoconductive switch and 4-N,N-dimethylamino-4'-N'-methyl-stilbazolium tosylate (DAST) organic electro-optic crystal and (right) their Fourier-transformed spectra. Dips correspond to water vapor rotational absorption lines [2]. The electro-optic generation shows an higher dynamic range.

Typical electric field amplitude waveforms and corresponding spectra of both generation mechanisms are illustrated in Figure 1.5. Through a spectroscopic point of view, THz-TDS directly measures the complex
refractive index of the specimen and accesses its complex permittivity without using the Kramers-Kronig relationship. Nevertheless, conversion efficiency is on the order of $10^{-4}$-$10^{-6}$, resulting in low average pulse power levels that range from nanowatt to microwatt.

Most of THz imaging research has been done replicating a TDS modified setup demonstrated by Hu and Nuss in 1995 [6]. Here, a complete time-domain waveform data set is acquired one pixel at a time, raster-scanning the target through a focalized THz beam: phases and amplitudes of any subset of frequency components of the transmitted pulse can be used to infer different types of information about the sample, as shown in Figure 1.6.

Figure 1.6 THz-TDS transmission images of a chocolate bar. (a) Variations in peak-to-peak amplitudes of the transmitted pulse. Here, the embossed lettering is only visible because of scattering effects at the stepped edges, whereas the almonds are clearly visible due to their stronger THz absorption. (b) Variation in transit time of the pulse, reflecting the accumulated phase of the field. The thickness of the sample is quite clear. Adapted from [6]
1.3 Time-of-flight measurement

Figure 1.7. (a) Reflected intensity THz image of a 3.5-in floppy disk. (b) Time-of-flight strip at y=15mm. Red and blue lines indicate positive and negative refractive index discontinuities (air-to-plastic, plastic-to-air). The metal hub gives rise to multiple reflections, which appear in the region beneath it. (c) Reflection from a single input pulse. Adapted from [7]

THz-TDS in reflection geometry can also acquire the positions and the magnitudes of the longitudinal dielectric-constant changes in a multilayer structure. When a single-cycle picosecond-long THz pulse is incident normally upon an object, the reflected waveform consists of a train of attenuated pulses, which originates from interfaces located at different positions in the sample. This scheme, named time-of-flight measurement, was first proposed in 1997 ([7]) and its application was demonstrated to nondestructive identification of defects in low-density, low-absorption space shuttle foam insulator, where other techniques failed [8]. As an example, Figure 1.7 shows a time-of-flight measurement of a 3.5” floppy disk.
1.4 Near-field techniques

Diffraction is the main limitation to the spatial resolution of all imaging systems. The minimum resolution is related to the spot diameter of the investigation beam, which can roughly be expressed as the wavelength multiplied by the f-number of the optics. As a consequence, spatial resolution achievable with terahertz waves lays in the sub-millimeter range, which is comparable to the resolution of the human eye.

One way to image sub-wavelength objects bypassing the diffraction limit consists in acquiring not only the propagating electrical field but also the evanescent waves that build up in a one-wavelength zone close to the sample under study. Like many of the cases already shown, some established optical techniques have been extended to the terahertz domain. Since its first demonstration in 1998 [9], terahertz sub-wavelength imaging has become one of the most active research areas in the terahertz community. The following discussion aims to give a brief overview of this rich and dynamic subject. We refer to established reviews on THz microscopy for details [10] [11] [12].

The simplest way to achieve a sub-wavelength resolution consists in using an aperture located within a wavelength from the object under investigation. Using this trick, only the non-diffracted field that enters the aperture contributes to the formation of the image, resulting in a resolution determined by the size of the aperture rather than the wavelength of the radiation [9].

Zhang and co-workers developed in 2000 the so-called dynamic aperture technique, which has no optical counterpart [13]. This architecture makes use of a modified version of THz-TDS imaging setup, where the optical
beam is focused to a much smaller area than the THz beam, so that the zone in which the THz radiation interacts with the photogenerated carriers is limited to a few tens of micrometers. In this way, the dimension of the optical pump governs the resolution.

The main disadvantages introduced by aperture-based methods includes the frequency cut-off of the metal waveguide created by the aperture and intrinsic power losses.

Another category of remarkable near-field techniques that exploit the scattering radiation from a metal probe, are the so-called aperture-less near-field microscopies (ANSOM). Two different implementations of this imaging method in the THz range were first reported by Kersting in 2003 [14] and by Planken in 2002 [15].

![Figure 1.8 Aperture-less near field techniques. (a) Vibrating tip method: the forward scattered field is directly measured in the far field via a lock-in scheme. (b) Collection in the near-field with a fixed tip: only the component of the electric field parallel to the optical probe is measured through EO sampling.](image)
The idea proposed by Kersting’s group makes use of a sharp vibrating metal tip in the near field of the sample (Figure 1.8 (a)). An incident THz beam is focused on its surface. The metal probe is moved back and forth along the surface normal with a small amplitude: scattered radiations underneath the tip is modulated at the vibration frequency and can be acquired by a lock-in detection. The resolution depends either on the distance between the tip and the sample or the edge dimensions. Recently, Keilmann’s group used the same principles to demodulate the continuous near field scattered off by an atomic force microscope tip locked at higher harmonics, reaching $\lambda/3000$ resolution [16].

Another strategy to measure directly the evanescent field instead of making it propagating, is shown in Figure 1.8 (b). Here, a stationary tip induces a change in the local electric field that can be measured through a non-standard electro-optic technique. In spite of the conventional TDS configuration, the terahertz beam counter propagates with respect to the optical pump pulse. The EO crystal is archly oriented so that the susceptibility tensor vanishes for components of the THz field polarized parallel to the crystal surface. In this way, only the normal component induced by the proximity of the metal tip to the crystal can be detected via the electro-optic effect. Here, the resolution is limited by the size of the tip and the optical spot diameter inside the non-linear crystal. Up to now, this is the only both spectroscopic and sub-micrometer-resolution imaging implemented with terahertz waves.
1.5 CW schemes

Figure 1.9 General raster-scan scheme using CW THz source

In parallel with developments of new multidisciplinary applications using pulsed time-domain techniques, tremendous progresses have also been made in continuous-wave THz technology. Any combinations of THz sources and detectors have been used in a universal raster-scanning scheme (Figure 1.9), where the transmitted, the reflected or the scattered optical power from a target is measured. Such incoherent imaging systems have been reported using pyroelectric detectors [17], superconducting Josephson junctions [18], room-temperature Schottky diodes [19], Golay cells [20] and cryogenically cooled bolometers [21].

Besides, although real-time imaging in the spectral region above 1 THz is still technologically challenging, a 25m stand-off video-rate (30 frames per second) imaging using a quantum cascade laser and a room-temperature commercial focal plane array microbolometric cameras has been demonstrated ( [22] [23] and [24]). The experimental setup and some imaging demonstrations are shown in Figure 1.10.
Figure 1.10 Experimental setup for imaging over a distance of 25m: a QCL, mounted in a pulsetube cryocooler, emits a beam that is collimated by an off-axis paraboloid mirror, before collection by a spherical mirror. In configuration (1), an object is placed 2 m before a spherical mirror, while in configuration (2), the sample is placed after a second off-axis paraboloid mirror. Visible frequency thumb print (d), and the same THz reflection image of the placed inside a paper envelope (c). Visible and THz image of a dried seed pod are shown in (d) and (e)-(f), taken respectively with configuration (1) and (2).
More recently, millimeter-wave imaging systems exploiting terahertz detection via nonlinear plasma excitation in graphene and nanowire based field-effect transistors have also been developed ([25] and [26] respectively). In this case, the incoming oscillating electric field, which is applied between source and gate electrodes, yields a modulation of both density and velocity of carriers, generating a constant voltage proportional to the incoming optical power. Scanning electron micrographs of these promising nanofabricated detectors are shown in Figure 1.11

![Figure 1.11](image)

**Figure 1.11** False-color scanning electron micrograph of nanowire-based FET (a) and graphene-based FET(b). (a) Source (S) and Drain (D) are located at opposite end of a InAs nanowire. (b) log-periodic circular-toothed antenna patterned between the source and gate of a Graphene-based FET. Imaging has been demonstrated at 0.3THz. Adapted from [25] and [26]

Unlike THz-TDS, the majority of continuous-wave imaging systems are sensitive only to the intensity of the electric field and therefore they lose the information encoded in its phase. Exceptions reported in literature includes coherent techniques based on photomixing [27], a pseudo-heterodyne method consisting in the mixing between the longitudinal modes of a multimode QCL [28], a recent implemented electro-optic harmonic sampling which makes use of a QCL phase-locked to a near-
infrared fs-laser comb [29] and, finally, the self-mixing approach, first extended in the THz regime by Dean et al [30].

Generally speaking, the use of CW sources has three main advantages over pulsed systems [31]:

i. Long-range imaging is possible at certain frequency windows where atmospheric attenuation is relatively low.

ii. Looking for a specific absorption feature, a narrow-linewidth CW source provides an higher spectral resolution.

iii. Simplicity and less expensive optical components.

However in some specific applications the full set of data acquired through THz-TDS is more suitable.

1.6 Imaging applications

Terahertz radiation has unique proprieties that have been exploited in a list of promising employments in a huge range of different research fields including medical and pharmaceutical science, security, industrial non-destructive evaluation, material science, art conservation and astronomy. What follows is a brief and incomplete list of applications.

The ability of THz radiation to cross common non-metallic materials (plastics, clothes, packaging, organic tissues) combined with the possibility for identification of distinct spectral fingerprints of most of illegal drugs and explosives has inspired many security applications, including weapon detection.

Due to its non-invasive nature, THz imaging research has also been applied in biological and medical diagnosis. In particular, because sub-
cutaneous skin cancers have different hydration levels from ordinary tissue, they can be identified exploiting the strongly absorption through polar molecules experienced by THz waves.

The water sensitivity of THz waves can also be availed to control the content of foods or the agricultural products without opening packaging.

Imaging exploiting T-ray has found applications in non-destructive measurements in pharmaceutics industry, such as monitoring of coating thickness and refractive-index profile of tablets [32]. Recently, the art conservation has taken advantages of terahertz technology to measure the thickness of covered layers of historical paintings, revealing previously sketched artworks.

A huge number of industrial applications can benefit from the transparency of THz radiation of packaging. Examples taken from the semiconductor area includes the evaluation of silicon solar cells, polymers and dielectric films or the estimation of material properties such as the mobility, the conductivity, the doping level, the carrier density and the plasma oscillations of a device.

Figure 1.12 illustrates a selection of the applications listed above.
1.6 Imaging applications

Figure 1.12. Selection of THz-imaging applications. (a) Absorption fingerprints of different explosives in the range 0.1-4 THz. (b1) Photograph of the sole of a shoe containing objects inside. (b2) Hidden objects revealed through TDS: a razor blade, a ceramic block and a small square of a plastic explosive material. Optical (c1) and THz (c2) image of an historical portrait. (d1) Three-dimensional thickness and (d2) refractive index extrapolation of a pharmaceutical tablet without opening the packaging. (e) Medical diagnosis demonstration of a carcinoma (From www.Teraview.com)
Chapter 2

Optical feedback

Studies about the intrinsic nonlinear dynamics of lasers date back to the seminal work of Haken in 1975 [33], who connected their mathematical description with chaotic turbulence seen in fluids. A plethora of complex phenomena appears when a semiconductor laser is subjected to optical feedback, i.e. back-reflected or scattered coherent radiation interact with the light already present in the cavity laser.

The chapter is organized as follows. The Lang-Kobayashi model is presented (2) and the steady-state solutions for carriers and fields are then discussed (2.2). The self-mixing (SM) scheme, the interferometric technique on which this work is based, is illustrated in paragraph 2.3, while we pay particular attention to general feedback regimes of classical semiconductor lasers in paragraph 2.4. Finally, the general proprieties of quantum cascade lasers are outlined in paragraph 2.5 and novel results about the effects of optical feedback in these sources are discussed in paragraph 2.6.
2.1 The model

The first theoretical effort at describing the effect of weak-to-moderate optical feedback in a laser diode was made in 1980 by Lang and Kobayashi (LK) [34], whose pioneering paper has motivated a great number of following investigations. Although their original model describes the time evolution of the electric field in a single (longitudinal) mode laser diode using a rate equations approach, it has also been extended to include multimode operations.

The system can be considered as a compound cavity (Figure 2.1), composed by the active region surrounded by two plane mirrors with (power) reflectivity $R_1 = R_2$, followed by an external cavity created by one facet of the laser and a target with a reflectivity $R_{ext}$, which is located at a distance L. We consider the electric field $E(t)$ as the product of the slowly varying envelope $E_0(t)$ and the rapidly oscillating term $e^{j\omega_0 t}$, centered on the solitary frequency $\omega_0$. Taking into account only one reflection from the external target, the field reenters the laser cavity after an external round-trip time $\tau = 2L/c$ and adds coherently to the laser field. Using the plane
wave approximation and the mean field limit, i.e. neglecting spatial variation of the field amplitude in the transversal plane and along the optical cavity, this process can be described via a modified version of the Lamb’s rate equation with the inclusion of a time-delayed term:

$$\frac{d}{dt} E_0(t)e^{i\omega_0 t} = \left[i\omega_q(N) + 1/2 (G(N) - \Gamma_0)\right]E_0(t)e^{i\omega_0 t} + \frac{k}{\tau_c}E_0(t-\tau)e^{i\omega_0(t-\tau)}.$$ 

The first term represents the possible difference between $\omega_0$ and the instantaneous frequency of the Fabry-Pérot resonator, given by $\omega_q = q\pi c/\eta l$, where $q$ is an integer and $\eta$ and $l$ are respectively the refractive index and the length of the active medium. A strong *ansatz* is made here: the changes in carrier density $N(t)$ caused by the changes in the electric field profile lead to variations of the refractive index due to plasma loading, which in turn affects the instant frequency of the laser. This mechanism, named *phase-amplitude coupling*, can be better explicated though the definition of a linewidth enhancement factor:

$$\alpha = \frac{4\pi}{\lambda} \frac{d\eta}{dN} \frac{dG}{dN}.$$  \hspace{1cm} (2.1)

The second contribution in the E-field equation arises from the effective amplification, given by the difference between the stimulated emission $G$ and the cavity losses $\Gamma_0$. The final term is the original addition made by Lang and Kobayashi, which accounts the effect of the time-delayed feedback coupled with the coefficient

$$k = \epsilon (1 - R_2) \sqrt{\frac{R_{\text{ext}}}{R_2}}.$$  \hspace{1cm} (2.2)
where the dimensionless parameter $\epsilon$ quantifies the spatial mode overlap between the back-reflected light and the lasing mode (typically $\epsilon = 0.01 - 0.5$) and $\tau_c$ is the laser cavity round-trip. This expression can be obtained by requiring the continuity of the electric field at the boundary of the two cavities and when the condition $R_2 \gg R_{ext}$ holds. When small perturbations of $\eta$, $G$, $\omega$ - due to variations in carrier density near the unperturbed laser threshold - are considered, LK equations can be rearranged as

$$\frac{d}{dt}E_0(t) = \frac{1}{2}(1 + i\alpha)G(N(t) - N_{th})E_0(t) + \frac{k}{\tau_c}E_0(t - \tau)e^{-i\omega_0 \tau}. \tag{2.3}$$

The rate equation for the carrier density completes the picture:

$$\frac{d}{dt}N(t) = -\frac{N}{\tau_e} - G(N(t) - N_{th})|E|^2 + \mu. \tag{2.4}$$

Here, the first and the second terms describe respectively the spontaneous and stimulated emission ($\tau_e$ is the carrier lifetime, $N_{th}$ is the carrier density at threshold), while $\mu$ quantifies the electrical injection.

### 2.2 Steady state solutions

We can get some physical insights considering the real phase and the amplitude of the field $E(t) = E_0(t)e^{i\varphi(t)}$ and searching steady-state solutions: $E_0(t) = E_0(t - \tau) = E_s$, $N(t) = N_s$ and $\varphi(t) = (\omega_s - \omega_0)t$.

After some algebra we can obtain steady-state solutions for frequencies, carrier density and optical power:

$$(\omega_0 - \omega_s) \tau = C \sin[\omega_s(\tau) + \arctan \alpha] \tag{2.5}$$
Steady state solutions

\[ N_{th} - N_S(\tau) = \frac{2k}{G\tau_c} \cos[\omega_0(\tau)\tau] \]  \hspace{1cm} \text{(2.6)}

\[ |E_s|^2 = \frac{1}{G(N_S - N_{th})} \left( \frac{\mu - N_S}{\tau_e} \right) \]  \hspace{1cm} \text{(2.7)}

where

\[ C = \frac{k}{\tau_c} \tau \sqrt{1 + \alpha^2} \]  \hspace{1cm} \text{(2.8)}

represents a dimensionless parameter which controls the dynamical behavior of the system. Owing to this analysis it is clear that the optical retroaction modifies the threshold carrier density and hence the gain and the emitted power of the laser.

\[ \begin{align*}
\text{Figure 2.2 (a) Round trip phase difference versus the instantaneous frequency for two feedback parameters } & C_1 \text{ (solid line) and } C_2 = 2C_1 \text{ (dashed line).} \\
& \text{(b) Modes (stars) and antimodes (triangles) sustained by the cavity.} \\
\text{Eq. 2.5 is a transcendental relationship owing to the dependence of the frequency of the laser on the external round trip } & \tau, \text{ and can be solved numerically. Figure 2.2 (a) shows the graphical solution of Eq. 2.5 for two different feedback parameters: many frequencies } \omega_k \text{ are sustained by the compound cavity and their number is controlled by the parameter } C. \\
& \text{Steady-state solutions can be found setting to zero the round trip phase of} \\
\end{align*} \]
the field $\Delta \varphi(\tau) = (\omega_0 - \omega_0) \tau = 0$. As can been seen, increasing the feedback or $\tau$, new solutions arise in pairs. This effect can be interpreted as the constructive (mode) and destructive (anti-mode) interference between the cavity and the reflected fields. This explanation is endorsed by the fact that these modes are separated in frequency by a free spectral range of the external cavity. However, in a relatively weak-feedback condition, it has been analytically shown that only one solution is stable of all the possible frequencies of the system (Figure 2.2 (b)), which corresponds to the maximum gain (and to the minimum threshold) mode.

From an experimental perspective, the estimation of the feedback parameter appears difficult because of the uncertainty on the coupling $\varepsilon$. To access $k$, one measures the effective difference between the current threshold in the presence and without the feedback, respectively $I_{th}^{sol}$ and $I_{th}$. This difference is proportional to $k$ via:

$$\Delta I = \frac{I_{th}^{sol} - I_{th}}{I_{th}^{sol}} = k \frac{\tau_p}{1 + GN_0 \tau_p}.$$  \hspace{1cm} 2.9

### 2.3 Self-mixing interferometry in semiconductor laser diodes

The study of optical feedback has led to a big amount of fundamental physical knowledge in photonics and, at the same time, has stimulated the development of practical applications, such as coherent echo detection, chaotic signal communications and self-mixing interferometry. In the latter, the laser source acts as a coherent detector, sensitive to changes in the amount of the back-coupled radiation and of its experienced phase-
shift, which is caused by the presence of an external target. Interferometric measurements exploit the so called weak-to-moderate regime, where the fraction of the perturbing field brought back in the laser cavity is in the range of approximately $10^{-8}$-$10^{-4}$ of the free-running field intensity. In these conditions, the mathematical analysis previously conducted can be simplified, making the assumption $k \ll \tau_c/2\tau_e$, and a straightforward expression for the emitted power can be derived:

$$P(\phi) = P_0 [1 + m F(\phi)].$$ \hspace{1cm} 2.10

Here, $P_0$ is the optical power without feedback, $m = 2k\tau_e/\tau_c$ is the modulation index, $\phi = \omega(\tau)\tau$ is the phase accumulated during the external round-trip by the electrical field and $F$ can be implicitly defined as $F(\tau) = \cos[\tau\omega(\tau)]$, whose periodic waveform strongly depends on $C$ through Equation 2.5.

Owing to the characteristics of the interferometric signal and the value of $C$, different regimes can be identified:

i. $C < 0.1$, very weak feedback regime. The function $F(\phi)$ resembles the cosine function of a conventional Michelson interferometer.

ii. $0.1 < C < 1$, weak feedback regime. The function $F(\phi)$ becomes distorted, and takes an asymmetrical shape that can be exploited to acquire information about the direction of the target displacement.

iii. $1 < C < 4.6$, moderate feedback regime. The function $F(\phi)$ becomes two-valued and the interferometric signal shows a sawtooth-like behavior with hysteresis. The modulation index shows an
experimental saturation when the amount of the back-coupled radiation is increased.

iv. \( C > 4.6 \), strong feedback regime. The \( F(\phi) \) becomes five-valued, and the system leaves the self-mixing regime and can experience mode-hopping, coherent collapse or routes to chaos.

Figure 2.3 shows more insight into the features of the self-mixing signal when the parameter \( C \) is varied to cover all the regions previously defined. Irrespective of the feedback regime, the modulation function shows a periodicity of \( 2\pi \). This means that if the external reflector is moved along the longitudinal direction, SMI signal reproduces itself every \( \lambda/2 \) displacement.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.3.png}
\caption{Calculated SMI modulation waveform as a function of the phase for different values of the feedback parameter: (a) \( C=0.1 \), (b) \( C=0.7 \), (c) \( C=3 \), (d) \( C=6 \). It is assumed \( \alpha = 6 \). Adapted from [35]}
\end{figure}

A small value of \( C \) (Figure 2.3 (a)) can be interpreted as a small coupling between the external and the internal cavity, which results in a symmetrical waveform very similar to a standard interferometer, with different power in the two arms. As the value of \( C \) approaches unity (Figure 2.3 (b)), the signal gets asymmetric and the discrimination of the
direction of motion of the target is possible with no need for two quadrature readings.
However, when $C = 1$ the system becomes bistable: it has three solutions for certain values of the phase, whereof one is non-physical. A stability analysis can be performed to identify which branch is stable. Let’s consider as an example Figure 2.3 (c), and suppose that the system is in the state $W$: when the phase is increased, it moves along the curve up to the point $X$, where it jumps down to point $X'$, which is located on the adjacent stable branch. Vice versa, if the system is placed in $X'$ and the phase is decreased, point $Y$ is reached, and an upper jump to point $Y'$ subsequently occurs. Therefore, in the moderate regime the interferometric modulation exhibits hysteresis and discontinuous step-like transitions.

An analytical expression for the hysteresis phase period $\phi_{Hyst}$ can be calculated finding in Equation 2.5 the phase where the derivative of the angular frequency diverges:

$$\phi_{Hyst} = 2 \left[ \sqrt{C^2 - 1} + \arccos(-C^{-1}) - \pi \right]. \quad 2.11$$

Surprisingly, the hysteresis duration depends only on the feedback coefficient $C$, showing an almost linear behavior for $C>3$.

Another change in the behavior of the system occurs at the threshold $C=4.6$: a new bifurcation takes place, and for certain values of the phase five solutions appear (Figure 2.3 (d)). Here, when the system reaches point $X$, two possible evolutions exist, since the system can jump down to two distinct stable points, namely $X'$ and $X''$. Similarly, when point $Y$ is reached with decreasing phase, the system can jump up to points $Y'$ or $Y''$. 
2.3 Self-mixing interferometry in semiconductor laser diodes

The intrinsic dynamics of the laser under investigation governs which of the jumps occurs.

One of the crucial points for the coherent phenomenon just reviewed is its applicability to a vast range of metrology measurements. Traditional quantities measured exploiting SM interferometry are displacements, velocities, vibrations, absolute distances and angles.

At the same time, owing to the complex relationship between carriers and photons involved, SMI has been employed to access physical laser quantities such as the linewidth, the coherence length and the alpha factor.

In traditional configuration the signal is read from a photodiode integrated in the packaging of commercial laser diodes. Another solution consists in monitoring the perturbation of the voltage across the device [36]. In fact, the voltage is related to the carrier density by the Schottky relation

\[ N = N_i \exp \left( \frac{eV}{2kT} \right), \]  \hspace{1cm} 2.12

where \( N_i \) is the intrinsic carrier density, \( k \) is the Boltzmann’s constant, and \( T \) is the temperature. When this expression is combined with equation 2.6 the following simple relationship is obtained:

\[ \Delta V(\phi) = -\Delta P(\phi) \frac{R_{th}}{\eta}. \]  \hspace{1cm} 2.13

Here we have denoted with \( R_{th} \) the diode differential resistance at threshold and \( \eta = dP/dI \) the efficiency of the laser diode. The physical meaning is therefore clear: when the laser power is increased by the optical feedback the change in the voltage across the junctions can be thought as a voltage drop that would result from an equivalent increase in the injection current in the absence of feedback. It is important to note that according to equation 2.13 the laser power and the junction voltage are in phase
opposition. These considerations do not take into account the change in the DC bias of the laser, which is related to the feedback parameter $C$.

The drawback of this voltage-based scheme is that the junction output is affected by a worse signal to noise ratio with respect to the optical power acquired via a photodiode.

Indeed, the differential resistance $r_{diff} = \frac{dV}{dI} = \frac{2kT}{e_{DC}}$ found across the laser diode carries Johnson noise, which is given - in rms current- by $i_n = \sqrt{\frac{4kT B}{r_{diff}}}$ where $B$ is the bandwidth of the electronic system; this quantity adds in quadrature to the shot noise coming from the bias current and the total noise results therefore greater than the electronic one which characterized a typical photodetector.

2.4 Regimes

The discussion above has been carried out having a special consideration for interferometric applications. Nevertheless, pioneering works focused on more fundamental questions linked to the dynamics of the laser and its stability against back-coupled radiation. Theory and experiments have been chasing each other for the last thirty years, trying to explain fascinating complexity, which arise from the optical feedback in semiconductor laser.

This has been summarized by the diagram illustrated in Figure 2.4, designed for the first time in the seminal paper of Tkach and Chraplyvy in 1986 ([37]).

At the lowest feedback levels, Regime $I$, the laser operates on a single external cavity mode that is originated from the solitary laser mode.
Owing to the phase of the field, the linewidth can be narrowed or broadened. In *Regime II* and depending on the distance from the external reflector, the mode splits into two near frequencies. This effect arises from noise-induced hopping between two modes of the external cavity. The transition into *Regime II* has been found to correspond to the instability at $C=1$, when multiple solutions of the steady state equation 2.5 exist.


**Figure 2.4** Diagram of coupling-strength (back reflection attenuation) versus external cavity length. Regime I: linewidth narrowing/broadening. Regime II: line splitting and mode hopping. Regime III: return to single mode. Regime IV: low frequency fluctuations and deterministic chaos: Regime V: external cavity mode. Adapted from [37].

In *Regime III* the laser counterintuitively returns to a single external cavity mode (the lowest linewidth mode) with constant power in time. As the
feedback is still increased, and independently of the length of the external cavity, the system undergoes a transition to a chaotic state known as coherence collapse: it enters Regime IV. This is characterized by a dramatically broadened optical and noise spectra, which contains many external cavity modes. The low current injection regime near the solitary laser threshold is also known as the low frequency fluctuations regime, so named from the irregular and slow power oscillation events caused by the competition of a great number of possible external frequencies, none of which is stable. The route to this chaotic state starts with a series of Hopf bifurcations, which are characteristic of a huge number of systems belonging to different areas of physics. The broadened spectrum appears continuous or spike-like depending on the length of the cavity, which can be greater or less than \( L_{\text{freq}} = c/2f^* \), where \( f^* \) is the cut-off frequency of the laser diode associated with high frequency modulation.

Still further increase in the optical feedback level results in a final transition to another single mode, constant intensity, and narrow linewidth regime (Regime V), namely external cavity mode. This regime can be reached only by uncoating the laser facet and is typically exploited in spectroscopic applications.

Applications of SMI phenomena are located in the lower left side of the diagram, whereas chaos has been proposed as a cryptography tool. SMI obviously requires coherence of the returning field addition to the in-cavity unperturbed field, while chaos can be also generated from incoherent coupling.
2.5 Quantum cascade lasers

Quantum cascade lasers are unipolar semiconductor devices in which laser action is achieved in intersubband states located in the conduction band. These levels arise from the spatial confinement of electrons in few-nanometer-thick quantum-wells of a heterostructure. QCLs derive its name from the multistage scheme used, in which each electron travels through an active region sequentially replicated tens of times and emits multiple photons. This unique propriety leads to internal quantum efficiency greater than one and intrinsic high-powers: hundreds of milliwatts in continuous mode and peak pulse powers in the range of Watts have been achieved. Although the first idea of using intersubband transitions and tunneling in a cascade structure to produce light amplification was proposed by Kazarinov and Suris in 1971 [38], only in 1994 the first QC laser was experimentally demonstrated at Bell Labs [39], after the refinement of molecular beam epitaxy and improvements over transport models in heterostructures. Many differences with classical diode laser are inherently related with the use of intersubband transition.

Energy space between subbands can be designed by engineering the thickness of quantum wells and barriers. Thus, lasing in a remarkably wide range (3-200 μm) has been proved. In this way, the emission wavelength is independent from the energy gap of the constituent semiconductors. Moreover, since the initial and the final states involved in the stimulated emission have the same curvature in the reciprocal space, the related joint density states is very sharp as in gas laser. This results in a narrow linewidth theoretically predicted [40] and experimentally
measured [41], as small as 100kHz, reaching the quantum-limited frequency fluctuations (Hz) when the system is stabilized [42].

In order to acquire a simplified physical picture of working principles of QCLs, the active region can be modeled as a four-level system (Figure 2.5 (a)). When an electric field is applied, electrons are injected by tunneling from the subband 4 into level 3, which acts as the upper level of the laser. Here, carriers undergo stimulated emission from 3 to 2 and then quickly relax to 1, typically via a designed non-radiative longitudinal optical (LO) phonon interaction.

Neglecting the thermal backfilling from level 2, we can write a set of rate equations for electron sheet density in the upper $n_3$ and the lower $n_2$ levels (43)]:

$$\frac{dn_3}{dt} = \eta \frac{J}{e} - \frac{n_3}{\tau_3} - G(n_3 - n_2)S,$$

$$\frac{dn_2}{dt} = \frac{n_3}{\tau_{32}} - \frac{n_2}{\tau_2} - G(n_3 - n_2)S,$$

where $\eta$ is the injection efficiency, $S(t)$ is the photon number, $G$ is the gain coefficient, $\tau_3$ ($\tau_2$) is the total electron lifetime of level 3 (2), $\tau_{23}$ the characteristic time of the transition 3 $\rightarrow$ 2.

An expression for the propagation gain $G$ is obtained requiring steady-state condition

$$G \sim \Gamma \frac{\omega}{\delta \omega} \tau_3 \left(1 - \frac{\tau_2}{\tau_{23}}\right) |z_{23}|^2.$$
Here, $\Gamma$ is the spatial overlap of the guided mode with the active module, $z_{23} = \langle 2|z|3 \rangle$ is the dipole matrix element of the transition obtained through the Fermi’s golden rule. The latter is proportional to the stimulated-emission cross-section and heavily depends on the overlap and the symmetry of the initial and final wavefunctions; $\delta \omega$ is the spontaneous emission linewidth of the Lorentzian-shaped transition.

From these considerations it can be inferred that a population inversion and a positive gain are achieved when $\tau_2 < \tau_{23}$. The traditional approach used in mid-infrared QCL has been to couple level 2 and level 1 via non-radiative optical phonon interaction, which occurs at very fast rate (0.2-0.3 picoseconds) with respect to picosecond electrons radiative transition in subbands.

Besides, due to its ultra-fast carrier dynamics comparable to photon rates, quantum cascade laser is the only semiconductor system belonging to A-class lasers (Arecchi’s classification). Therefore, it does not exhibit typical relaxation oscillations, showing an overdamped transient dynamics towards the steady state, which allows intrinsic modulation bandwidths up to several tens of gigahertz.

The development of QCLs below the Restrahlen band (energy below the optical phonon absorption band of polar semiconductor, $\leq 20$ meV) has been more challenging than the ones working in the mid-infrared, due to two main difficulties. Foremost, the mid-infrared strategy of fast depopulation of the lower lever through LO-phonon scattering is more strenuous at terahertz frequencies, since the photon energies are lower than the LO-phonon energies (36 meV for GaAs). Owing to the small distance between the previously defined level 3 and 2, the selective
depopulation of the sole lower state is difficult, as the upper laser state will be however coupled with level 1. In other words, the condition for the population inversion is hardly achievable, because $\tau_2$ is comparable to $\tau_{23}$.

![Diagram](image)

Figure 2.5. (a) General four-level scheme of QCL. Conduction-band diagrams for traditional terahertz design schemes: (b) chirped superlattice (c) resonant-phonon (d) bound-to-continuum. Two identical modules of each one are shown. The squared of the wavefunctions are plotted (upper and lower radiative in red and blue, respectively). Minibands are represented as gray shaded regions. Adapted from [44]

Second, as free carriers absorption scales proportionally to the square of $\lambda$, clever projects of waveguides are required to minimize the modal overlap with any doped semiconductor cladding layers.
Since the first demonstration in 2002, three different active structures have been proposed, namely chirped superlattice (Figure 2.5 (b)), bound-to-continuum (Figure 2.5 (d)), and resonant-phonon (Figure 2.5 (c)) to overcome these problems. Nevertheless room-temperature operation is still a big goal.

The chirped-superlattice (CSL) design exploits the formation of mini bands when nm-thick quantum wells are stacked together to create a superlattice [45]. The radiative transition can be designed to involve the lowest state of upper mini band and the top state of the next one, similarly to conventional inter-band transitions in laser diodes. In this way a population inversion is established since scattering of electrons between the tightly coupled states within the miniband is faster than the inter-miniband one. LO-phonons are indirectly involved in the electron gas cooling.

An evolution of CSL scheme consists in the bound-to-continuum approach [46]. Here, the upper radiative level is a bound defect state while the lower laser state and the depopulation remains the same. Consequently, the transition is more diagonal in real space: the upper-state lifetime and the injection efficiency increase to the detriment of oscillator strength.

Finally, resonant-phonon approach has been extended in the THz region ([47]) through a trick: the wave function of the lower radiative state is spread over several quantum wells via a tunneling mechanism, while the upper state remains localized. Thus, the spatial overlap with the injector, located a LO-phonon energy down, is spatially differentiated.
2.6 Quantum cascade laser against optical feedback

The study of the optical feedback has been traditionally conducted on bulk semiconductor lasers. Recently, many groups have extended these investigations on quantum cascade laser and increasing interest about unforeseen results has emerged.

The theoretical model underlying the physics of the optical feedback in QCL is still based on the Lang-Kobayashi approach.

One example, which confirms the applicability of such framework, is represented by the measure of the linewidth enhancement factor of QCLs trough self-mixing interferometry: a value near zero confirming pre-existing theories has been measured in Mid-IR [48] and THz [49] cascade structure (Figure 2.6). Moreover, it seems to increase when the driving current is raised. In turn, this fact has dramatic implications both with the linewidth of the laser (which scales with \((1 + \alpha^2)\) through the famous Schawlow–Townes formula) and with the dynamical behavior of these sources against the optical feedback. We will focus on the second issue.

![Figure 2.6](image-url). Measured alpha-factor as a function of the injection current for (a) a Mid-IR DFB QCL [48] and (b) a THz QCL [49]
Partially owing to this negligible value of $\alpha$, it is believed that QCLs operate in the weak feedback regime beforehand defined ($C < 1$). The reasons of this small coupling propriety can also be attributed to the long cavity length of QCLs (about 1mm). These combined effects reduce by a factor 100 the feedback parameter, in comparison with diode lasers for a given external cavity length.

Very recently, the effects of optical feedback on the dynamical behavior of QCLs have been numerically and experimentally studied [50], reporting an unexpected ultra-stability of such systems. In fact, it seems that when the optical feedback is increased, QCLs do not experience the onset of nonlinear phenomena (including mode-hopping, intensity pulsation and incoherent collapse) illustrated above (see Figure 2.4), typical of conventional laser diodes, but remain in a single longitudinal mode regime. Figure 2.7 illustrates the different behaviors of a quantum cascade laser and a laser diode: stable continuous emission characterizes the QCL with a feedback amount two order of magnitude higher than the critical value for which coherent collapse appears in laser diodes.
2.6 Quantum cascade laser against optical feedback

Figure 2.7 Experimental fast Fourier transform of signal detected by (a) the PD integrated laser diode and (b) the voltage modulation across a QCL device. Figure (b) illustrates the stable CW emission a QCL subjected a high optical feedback ($k = 7.5 \cdot 10^{-2}$). Figure (a) shows the coherence collapse in a LD, with $k = 1 \cdot 10^{-3}$. The inset displays the time-behavior of a laser diode in a coherence collapse regime. Adapted from [50]

Both the low value of the alpha factor and the photon to carrier lifetime ratio can theoretically explain these strong evidences: the critical value of the coefficient $k$ which causes CW instabilities, increases up to one order of magnitude, with decreasing $\alpha$ and with high photon to carrier lifetime ratio (Figure 2.8).

Figure 2.8 Numerical simulations of the effect of the LEF and the photon to carrier lifetime ratio on the critical feedback level. When $\alpha < \alpha_c$ the QCL enters the ultra-stable regime. Adapted from [50]
We conclude this discussion with a practical consideration. Due to the lack of detectors, the most suitable way to reveal the self-mixing signal consists in monitoring the change of the voltage drop across the device. An analytical expression which links carriers to photons in a cascade structure is not trivial in QCLs, therefore eq. 2.12, which works well for a laser diode, cannot be trivially applied to QCLs. Nonetheless, the general equation 2.13 can be supposed to hold.
Chapter 3

Experimental Setups

In this chapter experimental setups are presented. Paragraph 3.1 gives some details about the sources used: a commercial Mid-IR and custom THz quantum cascade laser. Paragraphs 3.2 and 3.3 describe the experimental schemes respectively in the Mid-IR and terahertz regime, through which imaging of various samples has been carried out without the use of an external detector. The two final sections illustrate some aspects related to the spot of the beam. An estimation about the beam waist (3.4) and the research of the focal plane (3.5) can be intrinsically accomplished through self-mixing.

3.1 Sources

Mid-IR QCL

One of the sources adopted in the experiment is a commercial quantum cascade laser (Alpes Lasers, mod. RT-CW-DFB-QCL), working at wavelength of 6.24µm (equivalently 48 THz, or 1600 cm⁻¹) at room temperature. Although it can operate in both continuous and pulsed mode, only the first configuration has been used in the experiment. The laser can be tuned from 1598 to 1609 cm⁻¹ by changing its temperature through a Peltier controller. This feature combined with the relative narrow nominal linewidth (5MHz) makes it suited for spectroscopic applications too.
Figure 3.1 Current-voltage (right) and light-voltage (left) characteristics of the QCL, for different working temperatures

Figure 3.1 shows the electrical and the optical proprieties of the device, for a wide range of temperatures. When stabilized at 10°C, the device exhibits a slope $dP/dI = 0.20 W/A$, a threshold current $I_{th} = 460 mA$ and a differential resistance of $dV/dI = 3 \Omega$. Figure 3.2 illustrates spectra for different temperature and driving current. As many other QCLs, it is characterized by a large divergence of the order of 40 -60 degrees.

Figure 3.2 Nominal spectra of the Mid-IR QCL for different driving currents and temperatures
3.1 Sources

The terahertz quantum cascade laser used emits at 79.1µm (3.9 THz). It is a modified version of the bound-to-continuum scheme, and its design was originally proposed by Losco et al. [51]. As it has been written before, this approach appears to be favorable in the THz domain, thanks to the diagonal character (in real space) of the optical transition, which facilitates a selective carrier injection.

The original feature of the device is the design of the active region: two identical optical transitions are expected from two upper laser levels, closely separated by about 1 meV in energy (see Figure 3.3).

The existence of these two upper states has two main consequences. First, it allows twice the number of electrons to be stored in roughly the same

Figure 3.3 Simulated conduction band structure of the 3.9THz-QCL under an average electric field of 1.5 kV/cm. The two upper-level lasers are depicted in green and red.
energy range, which comes out useful for high-current and high-temperature operations. On the other hand, the dipole matrix element of each transition is smaller (~ 5 nm) than typical for standard THz QCLs emitting at the same frequency.

A cascading structure with this property can be realized engineering the thickness of the barriers and the wells of a heterostructure. When the Schrödinger equation coupled with the Poisson equation is solved, the eigenfunctions and the respective energy levels are obtained. Figure 3.3 shows the calculated conduction band of the QCL, when an electric field is applied.

The device was grown by molecular beam epitaxy employing a GaAs/Al\text{0.15Ga\text{0.85}}As heterostructure on a nominally undoped GaAs substrate with a surface plasmon waveguide. A 500 nm thick heavily doped (3.0 × 10^{18} cm^{-3}, Si) layer defines the lower contact of the laser for the designed single plasmon waveguide. The active region is repeated 120 times and the growth ends with a heavily doped (5.0 × 10^{18} cm^{-3}, Si) 200nm-thick GaAs contact layer.

The layer thicknesses of the active module, starting from the injector, are 4.0/10.5/2.3/18.2/1.9/18.3/0.7/14.7/0.7/12.5/1.5/11.0/2.7/11.0/3.0/11.0 nanometers (Al\text{0.15Ga\text{0.85}}As barriers are written in bold).

Experimental current-voltage and light-voltage characteristics are shown in Figure 3.4.
Figure 3.4 Current-voltage (blue dots) and light-voltage characteristics (red dots) of the laser under investigation. The dimensions of the points correspond to the errors of the measurement.

Figure 3.5 Spectrum of the THz-QCL when driven at $I = 0.69 \text{A}$ measured via FTIR.

Moreover, the experimental spectrum acquired via a Fourier transform infrared spectrometer (FTIR) is presented in the Figure 3.5: a longitudinal
single mode is observed near the laser threshold, while multimode behavior is expected at higher driving currents.

Lasing at 3.9 THz with a threshold current density of 82 A/cm$^2$ at 5 K was demonstrated. The maximum output power is achieved near 400 A/cm$^2$, still 4-5 times above threshold. Lasing is observed also in pulsed mode up to 70 K, with a peak power level in excess of several milliwatts at 10 K.

These good performances and the high dynamic-range of the device indicate that both transitions are contributing in parallel to lasing.
3.2 Mid-IR Setup

The implemented system does not require the use of an external detector to image the sample, exploiting the Mid-QCL as source and detector of radiation. The self-mixing-based setup is depicted in Figure 3.6.

![Diagram of Mid-IR self-mixing-based imaging setup]

**Figure 3.6 Mid-IR self-mixing-based imaging setup.** L1: AR-coated aspheric lens. L2: positive meniscus ZnSe lens. NF: Neutral Filter. OSC: Oscilloscope. LA: Lock-in amplifier. C: Chopper. F: low-pass filter. QCL: Alpha Lasers mod. RT-CW-DFB-QCL. Optical path=1m

Specifically, the mid-infrared quantum cascade laser, stabilized at 283K, is driven at a constant current $I = 528\text{mA}$ by a high compliance current controller. As known from the literature, the laser is more sensitive to optical feedback near its threshold; thus, in order to maximize the interferometric signal, we work with a driving current value 6% larger than the threshold.
The probe beam is first collimated by an anti-reflective coated aspheric lens with high numerical aperture (NA = 0.56) and a focal length of 4mm, and then focused on the specimen by a positive meniscus Zinc selenide (ZnSe) lens of nominal focal length \( f = 50.8 \text{ mm} \). The optical path so formed has a length of 1m.

A variable neutral filter allows adjusting the effective optical feedback reflected off the target surface and refocused upon the front facet of the QCL. In this way the feedback parameter \( k \) defined before can be directly controlled.

The coherent mixing of the reflected beam with the optical field inside the laser cavity produces voltage changes on the laser terminals at the current controller, which are revealed as modulation fringes across the device. To reveal the information contained in this small signal (3-4 mV) with respect of the bias voltage (7V) a lock-in technique is implemented. In more detail, the laser beam is modulated at the frequency of 1 kHz via a mechanical chopper set along the optical axis. The voltage difference is detected subtracting the DC contribute via an AC-coupling. The resulting signal is sent to a lock-in amplifier with time constant \( T_c \), triggered by the frequency of the chopper. At the same time, the signal is also acquired by an oscilloscope in order to double-check measurements. Besides, the electronic chain includes a pre-amplifier with a variable gain, which is set at 40dB (x100) and a low-pass filter characterized by a cut-off frequency of 100KHz. As known, this filter improves the signal to noise ratio, cutting the high-frequency noise.

The sample is raster scanned in the x-y directions through a computer-controlled motor. In this configuration, a trade-off between the acquisition
time and the SNR exists. Reasonable values for the scan velocity along the x-axis ranges from 0.1 to 0.5 mm/s, with steps of 250μm in the y-direction.

### 3.3 THz Setup

![THz Setup Diagram](image)

**Figure 3.7 THz Self-mixing-based imaging setup.** PR: parabolic reflector. OSC: Oscilloscope. LA: Lock-in amplifier. C: Chopper. F: low-pass Filter. QCL: CW Quantum cascade laser, Optical path=0.5m

The setup discussed in paragraph 3.2 can be easily modified to work with the terahertz quantum cascade laser as a source (Figure 3.7). Here, the QCL is mounted on the cold finger of a continuous-flow cryostat fitted with a polymethylpentene (TPX) window and kept at a heat sink temperature of 15 K. The THz QCL beam is collimated using a 2 inch f/1 = 50 mm 90° gold-plated off-axis paraboloidal reflector and focused by a second identical reflector at normal incidence upon the remote target.
Because of the high divergence of the laser, the beam is expected to cover the entire surface of the mirror. Therefore, in the optical path where the beam is collimated, its diameter is estimated to be of some millimeters. The radiation was then coupled back into the laser cavity along an optical path of about 0.5 m.

In our experiments, the THz QCL was driven at a constant current $I = 700$ mA for CW mode operation. The electronic chain used is the same described above. Again, the effect of feedback - SM signal - was measured as voltage modulation across the terminals of the device.

As usual in terahertz experiments, a pre-alignment of the apparatus is realized using a He-Ne source, which is focalized on the facet of the quantum cascade laser and follows the same optical path experienced by the THz beam. The alignment’s beam is then switch off, because it has been demonstrated that its presence strongly affects the voltage across the QCL.

### 3.4 Beam waist

As illustrated in chapter one, the dimension of the probe spot is one of the most important figures of merit, which governs the spatial resolution of an imaging system. The measurement of the beam waist $\omega$ can be accomplished via the known knife-edge method: a plate with a sharp edge is moved along the direction of interest $\gamma$, while a detector acquires the spatial integrated power. It can be demonstrated that, assuming the beam to be Gaussian with intensity $I = I_0 e^{-\frac{2x^2}{\omega_x^2}} e^{-\frac{2y^2}{\omega_y^2}}$, the total power can be expressed as a function of the coordinate of the plate edge, say $x$, as

$$P(x) = \frac{P_{Tot}}{2} \left[ 1 - \text{erf} \left( \frac{\sqrt{2}x}{\omega_x} \right) \right],$$
where $P_{\text{tot}}$ is the total power and $\omega_x$ is the $1/e^2$ radii of the beam in the $x$ directions. Instead of using an external detector, one can access the same information in reflection geometry, exploiting the change in the DC voltage across the laser, which is proportional to the optical power reflected.

Figure 3.8 reports the measurements of the beam waist of the THz radiation along $x$ and $y$ direction, both using a detector and through optical feedback. The beam appears elliptical with $\omega_x \approx 330\mu m > \omega_y \approx 150\mu m$, and the values obtained by the two methods agree.

**Figure 3.8** Knife-edge method exploited to measure the beam waist of the THz beam along the $x$ (a) (b) and $y$ directions (c) (d). Signal in transmission (b) (d) and reflection configuration measured through SMI (a) (c)
3.5 Beam focusing

As any other THz imaging system discussed before, one of the main experimental problems is related to the focusing of the probe beam. In principle, the surface of the sample under investigation must be in the focal plane of the final lens. Deviations from this condition result in the decrease of the spatial resolution. We can make some quantitative considerations recalling some arguments of physical optics.

Supposing the beam profile to be Gaussian (Figure 3.9), the target can be assumed to be located in the focal plane if its deviations from the latter are less than the Rayleigh length, defined as

\[ z_R = \frac{\pi \omega_0^2}{\lambda}. \]

Here, \( \lambda \) is the wavelength of the radiation and \( \omega_0 \) is the beam waist, that is the radial size of the beam in its narrowest point. When we use the estimations of the beam waist acquired via knife-edge method, \( \omega_0(\lambda = \)
Beam focusing

6.25 \mu m) = 35 \mu m, and \omega_0(\lambda = 79.1 \mu m) = 350 \mu m, we obtain z_{R, MidIR} = 615 \mu m and z_{R, THz} = 4.8 mm.

Figure 3.10 Technique to find the correct focus position through self-mixing. DC signal measured across the QCL versus the longitudinal position of the target. In blue the experimental points, in red the Gaussian fit.

Self-mixing scheme offers a simple strategy to verify if the beam is correctly focalized on the target. The solution consists in monitoring the DC SM signal while the target is sequentially moved along the longitudinal axis: the maximum in the signal corresponds to the maximal optical intensity that couples back in the laser cavity. Figure 3.10 shows an example of this procedure for the Mid-IR laser; the width of the peak inferred from a Gaussian fit is w = 1000 \pm 180 \mu m which is comparable to 2z_{R, MidIR}.

A simple semi-quantitative model based on ray optics supports the validity of this approach. Let’s consider the target located at a distance d from the focal plane f_1 of the final lens of the optical system. A collinear
ray placed at position $y_i$ from the optical axis, after being reflected back from the target, returns in a radial position $y_i'$, which is different from $y_i$ (see Figure 3.11)

![Figure 3.11 Self-focusing scheme offered by self-mixing configuration](image)

If $d$ is big enough, the radiation cannot be coupled back into the laser cavity. Using the ABCD matrix formalism we can calculate the radial coordinate of a ray - which has an initial position $y_i$ before the final lens 1 $(\theta_i = 0)$ - after travelling through the complete optical chain again:

$$
\begin{pmatrix} y_i' \\ \theta_i' \end{pmatrix} = \mathcal{S}(f_2) \cdot \mathbb{L}(f_2) \cdot \mathcal{S}(L) \cdot \mathbb{L}(f_1) \cdot \mathcal{S}(d + f_1) \cdot \mathbb{M} \cdot \mathcal{S}(d + f_1) \cdot \mathbb{L}(f_1) \cdot \begin{pmatrix} y_i \\ \theta_i \end{pmatrix}
$$

where

$$
\mathbb{L}(f) = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}, \quad \mathcal{S}(x) = \begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix}, \quad \mathbb{M} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
$$

are the matrix operators for, respectively, a thin lens of focal $f$, a free-space propagation of a distance $x$, and a mirror reflection. Assuming $\theta_i = 0$ and the onset of parameters used in the mid-IR setup, a linear relationship is obtained: $y_i \cong 10d[\text{mm}] \cdot y_i[\text{mm}] \mu\text{m}$. For $d = 1\text{mm}$ and $y_i$ ranging from 0 to
25\text{mm} the dimensions of the back coupled spot can result two times greater than typical QCL facet dimensions (10x100 \text{\mu m}).
Chapter 4

Imaging Results

This chapter is devoted to presenting the experimental results obtained through self-mixing approach, used as an imaging tool. Paragraph 4.1 and 4.2 illustrate imaging outcomes achieved with mid-infrared and a terahertz setup. The role of the phase in these configurations is then discussed. The chapter ends with a focus on some aspects related to the visibility of the interferometric fringes. The non-conventional behaviour of the source against the amount of the optical feedback is reported in section 4.3

4.1 Mid-IR imaging results

Figure 4.1 (a) Optical microscope image of a lithographic mask. (b) Self-mixing voltage across the Mid-IR QCL, obtained by scanning the same sample in the x direction, revealing jumps in the DC voltage of the laser which correspond to the two different materials
The most intriguing peculiarity of the self-mixing-based imaging is its sensitivity to the phase of the field. Two pieces of information can be discerned from such a coherent scheme:

i) The incoherent amount of the radiation scattered or reflected from a sample, codified in the parameter $k$ of the Lang-Kobayashi model.

ii) The change in the phase of the field due to a modification of the optical path experienced by photons.

While the first contribute influences both the waveform of the interferometric signal and the DC-voltage of the laser, the second governs the periodic modulation of the voltage across the device. Unlike other coherent imaging systems reviewed in chapter 1, these two terms cannot be easily extracted separately, but they mix together. In order to illustrate these concepts, let’s consider a flat sample constituted by a series of stripes made of two materials with different reflectivity. Inter alia, the term flat indicates that any roughness has much smaller dimensions than $\lambda$.

One practical example of such structure is a mask employed in optical lithography, which is made up of alternating bands of chromium ($R_{Cr}(\lambda = 6\mu m) = 0.58^2$) and quartz ($R_{SiO_2}(\lambda = 6\mu m) = 0.03$) with different widths. Assuming that the sample is not tilted, when its image is acquired employing the setup discussed in paragraph 3.2, only the DC signal will give (incoherent) information about its composition. This is shown in Figure 4.1, where the highest value of the voltage corresponds to the

\[ \text{Figure 4.1} \]

---

material with higher reflectivity. However, when the mask is purposely rotated along the yaw axis, interferometric modulations superimposed on the dc signal arise from the effective longitudinal displacement seen by the sensor. Figure 4.2 shows SMI signal versus the x and y coordinates, when a +0.125° (a), 0° (b), -0.125°(c) tilted mask is raster-scanned. Interferometric fringes with typical shapes of the weak regime are clearly evident in the area where only Chromium is present (x > 5).

On the other hand, the phase modulation overlies the greater voltage variations, which arise from the reflectivity discontinuities of the sample. Besides, because of the opposite effective longitudinal motion of the target in (a) and (c), the slopes of the modulation waveform are reversed.

Finally, we can infer the tilt angle of the specimen and three-dimensional information. Recalling that every fringe is equivalent to a $\lambda/2$ displacement along z, the angle is easily obtained counting the number of the fringes N (10 in our case)

$$\theta = \arccos\left(\frac{N\lambda/2}{x}\right) = 0.115^\circ,$$

where x=15mm indicates the path of the scan along the x direction. The precision of this measurement can be calculated by error propagation, considering an error of one fringe ($\lambda/2 \sim 3 \mu m$) in the longitudinal coordinate of the target:

$$\delta \theta = \frac{1}{\sqrt{1 - \frac{N\lambda/2}{x}}} \frac{\lambda/2}{x} \approx \frac{\lambda}{2x} = 0.002^\circ.$$
The value so measured is in good agreement with the nominal value imposed by the mechanical stage, and the difference can be attributed to the tilt in the $y$-direction, evident in Fig. 4.2. (b)
Figur 4.2 Three-dimensional image of the lithographic mask acquired with three different tilts (a) yaw=0.125° (b) yaw=0° (c) yaw=-0.125°. For $x > 5$, the sample is made only by Cr
Figure 4.3 Effect of optical feedback levels on the phase sensitivity in QCL based imaging. The intensity of SM signal is in mV and is shown via a false-color scale. (a) Reflectivity image at a high level of optical feedback. (b) Phase information (SM fringes) is retrieved only by reducing the feedback level. At the bottom, a single trace acquired at a fixed y position.

The dynamical behavior of the QCL subjected to optical feedback has dramatic consequences on the capability of retrieving information about the phase of the field. In fact, we have experimentally demonstrated that fringes gradually disappear as the optical feedback (k) is increased.
4.1 Mid-IR imaging results

Figure 4.3 shows a representative reflection image as acquired by raster-scanning a specimen. In the case, the sample is part of a compact-disc front surface, consisting of a label of aluminum deposited on a diffusive substrate of polycarbonate. The target is again purposely tilted (yaw = 0.125°) to acquire the phase profile. Fringes associated with the spatial phase information are not visible in Figure 4.3 (a), and can be retrieved only by reducing the optical feedback, as shown in Figure 4.3 (b). A possible explanation is presented in paragraph 4.3.

Moreover, due to a scattering mechanism at interfaces, the morphology of a sample can be surveyed with the use of the DC signal only. Figure 4.4 illustrates the demonstration: a one-cent euro coin is imaged. Fringes are not visible because of the high optical feedback.

![Mid-IR image of a one-cent euro coin](image)

**Figure 4.4** Mid-IR false-color image of 1-cent coin using SM, which demonstrates the capability of the system to resolve morphology of the sample.
4.2 THz imaging results

Figure 4.5 shows the image acquired though the self-mixing scheme, which makes use of a terahertz quantum cascade laser as the source and the detector. Here, the specimen is a dime (one tenth of a United States dollar), which has been raster-scanned in the x-y plane. The gray color scale of the picture codifies the value of the self-mixing signal. The contrast of the coin’s edges is attributed to the scattering mechanism of the radiation at interfaces. Thus, the main contribute to the image formation springs out from the DC term of the signal. In this case interferometric fringes can be identified where the coin turns out to be flat and their presence is due to the tilt of the sample. As expected, the spatial resolution is in the sub-millimeter range. Moreover, this imaging scheme is characterized by a signal to noise ratio greater than 35dB and a relatively high dynamic range (DR = 45dB).

Figure 4.5 THz false-color self-mixing image of a dime coin. Fringes arises from the tilt of the sample
Figure 4.6 Optical (a) and 3.8 THz (b) image of a leaf, showing the capability of the system to reveal the presence of water, which is strongly absorbed by terahertz radiations.
Figure 4.6 shows another example of application of the implemented imaging system. Here the sample under investigation consists of a leaf, fixed upon a substrate of SiO₂. Owing to the strong absorption coefficient of water at terahertz frequency ($\alpha = 800 \text{cm}^{-1}$ at 3.8 THz) its water content can be mapped. Although not yet quantitative, this can be considered as the first demonstration towards spectroscopic imaging through self-mixing at terahertz frequencies.

4.3 Fringe visibility

As anticipated in paragraph 4.1, the amplitude of the interference fringes is a function of the feedback parameter k. In conventional laser diodes...
subjected to a weak feedback, the amplitude monotonically grows when the feedback is increased until a saturation value is reached [52].

A different behavior characterizes quantum cascade lasers as shown in Figure 4.7. Here, the magnitude of the fringes is acquired by moving a metallic (Al) target along the longitudinal direction. The feedback parameter is indirectly measured through the equation

\[ k = \left( \frac{l_p - l_{ps}}{l_{ps}} \right) \frac{\tau_c}{2\tau_p} \]

where \((l_p)_{th}\) and \((l_{ps})_{th}\) are the pump intensities at threshold with and without the feedback, \(\tau_c\) is the laser cavity round-trip (for a 1mm long cavity \(\tau_c = 35\text{ps}\)), \(\tau_p\) is the photon lifetime, assumed equal to 10ps.

Observing Figure 4.7 it can be ascertained that for \(k < k_c := 1.3 \cdot 10^{-3}\) the amplitude \(dV\) increases with the feedback (region a), reaching a plateau for \(k > 0.6 \cdot 10^{-3}\) (region b); for \(k > k_c\) (region c) fringes gradually disappear.

Figure 4.8 displays different shapes of the modulation signal when \(k\) is increased over the critical value: the more the feedback is increased, the more the signal seems to get symmetric resembling the geometry of the very-week regime. When \(k \approx 2.9 \cdot 10^{-3}\), the signal merges with the noise of the system, and no information can be got from the phase profile.

From these considerations, it is evident the strong relationship between the imaging system implemented and the more fundamental aspects related to optical feedback in QCLs. Therefore, more investigations are necessary to gain insight into these dynamics.
Figure 4.8 Waveforms of the interferometric signal from $k=0.0013$ to $k=0.0029$. When the feedback is increased the amplitude of the fringes decreases and the signal gets symmetric.
Finally Figure 4.9 illustrates the behavior of the DC voltage as a function of the feedback parameter $k$.

The measurement has been accomplished inserting a series of Polymethylpentene (TPX) filters along the optical axis and measuring the change in the DC voltage drop across the device, originated by the feedback from a metallic target. The upper axis reports the optical power without the feedback, measured behind the filters. The voltage linearly increases with the feedback, showing saturation for high feedback. This curve can be used to calibrate the sensor to acquire quantitative information about reflectivity of unknown targets.
Chapter 5

This chapter has been omitted in the online version of this work because the results which it reports are under publication.
Conclusions and future work

This thesis has discussed the design, the implementation and the characterization of two imaging apparatus, in the mid-infrared and the terahertz regime, based on self-mixing interferometry with quantum cascade lasers.

Some aspects related to the heterodyne principle of this scheme have been investigated, with a particular focus on the phase retrieval associated with different information content of the sample.

Moreover, some differences between the behaviour of quantum cascade lasers and conventional laser diodes under optical feedback have emerged. It seems that the recently demonstrated ultra-stable regime in QCL subjected to feedback influences the fringe visibility of the self-mixing signal.

Finally, preliminary results about the measurement of carrier-density profile have been drafted, although more research has to be carried out in order to compare the experiments to the underlying theory.

Theoretically, owing to the property of coherence of the system, SMI could be used to implement a holographic scheme in which the modulus of reflectivity of the target is kept constant while its phase could change.
From another point of view, an interesting evolution of this work could be the implementation of a complete terahertz three-dimensional imaging, borrowing tomographic approaches traditionally belonging to other regions of the electromagnetic spectrum, such as computed tomography, synthetic aperture or Kirchhoff migration tomography [11].

Another interesting area to investigate resides in the capability of the self-mixing to derive quantitative spectroscopic information from the sample. In such a way, the laser could be wavelength-tuned with the purpose of studying samples with different absorption coefficients, exploiting the intrinsic narrow linewidth of quantum cascade lasers.

Moreover more efforts are required to confirm the preliminary observations about the possibility of the system to map a charge distribution of an optically pumped semiconductor. Other semiconductors (GaAs, InAs) could be used as substrates, and more work must be accomplished in order to estimate the sensitivity of the SM method and, at the same time, to exclude other possible causes of modifications of the optical proprieties (thermal effects above all).

In the end, future work will endorse studies about non-conventional material behaviours through self-mixing interferometry. These will include investigations on complex optical constants of metamaterials, i.e. structures whose optical proprieties can be artificially engineered, and the exotic dynamics associated with them, such as negative refraction, and hyperbolic behaviour.
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